What happens to a city once the people are gone? Often, it lies buried in the earth waiting for discovery. Once it is excavated, its roads, buildings, sewers, and drains provide clues to the habits and ingenuity of the people who lived there. Fragments of pottery, sculptures, paintings, and toys offer glimpses into the society’s values, beliefs, and daily lives.

Archaeologists discover and decipher these clues from the past. Through painstaking digging and sifting through the remains of ancient cities, archaeology has given us a remarkable portrait of the world as it once was and the wonders of the past.

- **Heavy Equipment Operators** (page 211) clear the land, dig, and move dirt, debris, rock, and water to uncover archaeological ruins.

- **Archaeologists** (page 229) trace the histories of ancient civilizations by studying maps, artifacts, and the writings of ancient people. Archaeologists must be able to catalog items and draw conclusions from the clues to the past that they find.
Use the table for Questions 1–3.

1. Find the volume in cubic feet of the Great Pyramid of Khufu. Use the formula for volume of a pyramid: \[ V = \frac{1}{3}Bh \], where \( B \) = area of the base and \( h \) = height.

2. The Lighthouse of Alexandria was toppled by an earthquake in the fourteenth century A.D. Approximately how long did it stand?

3. The largest pyramid ever built is not the Pyramid of Khufu. It is the Quetzalcoatl, located in the ancient city of Cholula in modern-day Central America. This monument, about 177 ft tall, has a volume estimated at about 116.5 million ft\(^3\). About how long would it take to walk around it? Explain how you figured it out.

**Chapter Investigation**

Archaeologists often build three-dimensional models as an aid to understanding how an artifact, monument, or building may have looked at the time it was built. Scientists use ancient writings and their knowledge of the customs of the people to make educated guesses about the features and functions of structures that no longer exist.

**Working Together**

Choose an ancient structure for further research. Gather data about the measurements and known features of the structure. Then make a scale model or drawing of the structure. Using your data, estimate the exterior surface area and volume of the structure. Use the Chapter Investigation icons to guide your group.
Are You Ready?

Refresh Your Math Skills for Chapter 5

The skills on these two pages are ones you have already learned. Stretch your memory and complete the exercises. For additional practice on these and more prerequisite skills, see pages 654–661.

In this chapter you will solve problems involving perimeter, circumference, and area. It may be helpful to review a few of the basic formulas.

**Perimeter, Circumference, and Area**

You can use these formulas to find the perimeter and area of a rectangle, the circumference and area of a circle, and the perimeter and area of a triangle.

**Examples**

\[
P = 2b + 2h \\
= 2(7) + 2(5) \\
= 14 + 10 \\
= 24 \text{ cm}
\]

\[
A = b \times h \\
= 7 \times 5 \\
= 35 \text{ cm}^2
\]

\[
C = 2\pi r \\
\approx 2 \times 3.14 \times 4 \\
\approx 25.12 \text{ cm}
\]

\[
P = a + b + c \\
= 6 + 8 + 10 \\
= 24 \text{ cm}
\]

\[
A = \pi r^2 \\
\approx 3.14 \times 4^2 \\
\approx 3.14 \times 16 \\
\approx 50.24 \text{ cm}^2
\]

Find the perimeter or circumference and the area of each figure. Round answers to the nearest hundredth if necessary. Use 3.14 for \(\pi\).

1. [Rectangle]
   - Length: 6.5 cm
   - Width: 12 cm
   - Perimeter: \(2(6.5 + 12) = 2(18.5) = 37 \text{ cm}\)
   - Area: \(6.5 \times 12 = 78 \text{ cm}^2\)

2. [Triangle]
   - Base: 3 m
   - Height: 5 m
   - Perimeter: \(3 + 5 + \sqrt{3^2 + 5^2} = 3 + 5 + \sqrt{9 + 25} = 3 + 5 + 10 = 18 \text{ m}\)
   - Area: \(\frac{1}{2} \times 3 \times 5 = 7.5 \text{ m}^2\)

3. [Triangle]
   - Sides: 4 ft, 4 ft, 5.66 ft
   - Perimeter: \(4 + 4 + 5.66 = 13.66 \text{ ft}\)
   - Area: \(\frac{1}{2} \times 4 \times 4 = 8 \text{ ft}^2\)
The probability of an event can be expressed as a ratio:

\[ P(\text{any event}) = \frac{\text{number of ways an event can occur}}{\text{total number of possible outcomes}} \]

**Example**  
You toss a number cube. What is the probability that it will show an even number?

Number of ways to show an even number: \{2, 4, 6\} = 3  
Total number of possible numbers: \{1, 2, 3, 4, 5, 6\} = 6

\[ P(\text{even number}) = \frac{3}{6} = \frac{1}{2} \]

**Find the probability of each event.**

10. tossing a “heads” on a coin  
11. tossing a 3 on a number cube  
12. tossing a number less than 5 on a number cube  
13. picking a “diamond” from a standard deck of cards  
14. picking a “5” from a standard deck of cards  
15. picking a “jack of clubs” from a standard deck of cards  
16. picking a red marble from a bag of 8 blue marbles and 7 red marbles  
17. picking a brown sock from a drawer of 12 black socks and 3 brown socks
Imagine you are an archaeologist of the 25th century, and you have discovered your room, or another room in your home, looking exactly as it does today! List what you would find there. Describe how you would measure the contents. Then sketch the room as it would look when seen from above. Use a ruler and graph paper. Show all furniture and any rugs or other features you would see. Make your drawing as accurate as you can.

**BUILD UNDERSTANDING**

**Measurement** is a process we use to find size, quantities, or amounts. When you make measurements, you can use either *customary* or *metric* units. We can measure to varying degrees of accuracy. Different instruments are used to make different measurements.

The *compass* is used for drawing curved lines and circles. The *protractor* is an instrument for measuring and drawing angles. *Steel scales*, or rules, measure length.

Tool-and-die makers use *calipers* and *micrometers* to make precise measurements. Outside calipers are used to transfer the measurement of an object to a scale or drawing. Inside calipers are often used to measure diameters of objects. Micrometers are used to measure length and/or thickness.

The *precision* of a measurement is related to the unit of measure used. The smaller the unit of measure, the more precise the measurement. The *greatest possible error* (GPE) of any measurement is $\frac{1}{2}$ the smallest unit used to make the measurement.

**Example 1**

**ENGINEERING** An engineer is using a steel scale. The smallest markings on the scale are $\frac{1}{64} \text{ in.}$ What is the GPE of any measurement made with the scale?

**Solution**

Find half of $\frac{1}{64} \times \frac{1}{2} = \frac{1}{128}$. The GPE is $\frac{1}{128} \text{ in.}$

Measurements are often made in order to compare quantities. To compare quantities in the same unit, such as the length of a table in a drawing with the length of the actual table, you are using a ratio. A *ratio* is a quotient of two numbers that compares one number with the other.
There are three different ways to write a ratio. The order in which the terms appear is important. Each form below can be read “six to eleven.”

<table>
<thead>
<tr>
<th>analogy form</th>
<th>fraction form</th>
<th>word form</th>
</tr>
</thead>
<tbody>
<tr>
<td>6:11</td>
<td>( \frac{6}{11} )</td>
<td>6 to 11</td>
</tr>
</tbody>
</table>

For both customary and metric measures, you divide to change from a smaller unit of measure to an equivalent larger unit.

**Example 2**

Change 17 ft to yards.

**Solution**

3 ft = 1 yd

Divide 17 by 3 to find how many yards are in 17 ft.

\[
17 \div 3 = 5\frac{2}{3}
\]

So, 17 ft = \( 5\frac{2}{3} \) yd.

Use multiplication to change from a larger unit of measure to an equivalent measure in a smaller unit.

**Example 3**

Change 3.426 kg to grams.

**Solution**

1 kg = 1000 g

Multiply 3.426 by 1000 to find how many grams are in 3.426 kg.

\[
(3.426)(1000) = 3426
\]

So, 3.426 kg = 3426 g.

When you write ratios involving measurements, it is sometimes necessary to rename measurements using **like units**.

**Example 4**

Write the ratio of measurements 3 in. to 20 ft in lowest terms.

**Solution**

\[
\frac{3 \text{ in.}}{20 \text{ ft}} = \frac{3 \text{ in.}}{240 \text{ in.}}
\]

\[
\frac{3}{240} \div 3 = \frac{1}{80}
\]

The ratio of measurements is 1 to 80.
A ratio that compares two different quantities is called a rate. When you compare a quantity to one unit of another quantity, you are finding the unit rate. The unit price of an item is its cost per unit. Consumers can determine which of two items is the better buy by comparing the unit prices.

**Example 5**

**COST ANALYSIS** A 10-oz box of Cat Cravings costs $1.80. A 16-oz box of Kitty Yummies sells for $2.56. Which box is the better buy?

**Solution**

Write a ratio of price per weight for each product to find each unit price. Then compare prices.

\[
\frac{1.80}{10} = \frac{0.18}{1} = 0.18 \quad \text{Cat Cravings}
\]

\[
\frac{2.56}{16} = \frac{0.16}{1} = 0.16 \quad \text{Kitty Yummies}
\]

Kitty Yummies costs $0.16 per oz. Cat Cravings costs $0.18 per oz. Kitty Yummies is the better buy.

**Try These Exercises**

Change each unit of measure as indicated.

1. 2 gal to cups
2. 162 in. to yards
3. 3.5 L to milliliters
4. 6.25 km to meters

Write each ratio in lowest terms.

5. 12 m to 30 m
6. 9 yd : 4 ft
7. \(\frac{135\, \text{g}}{15\, \text{g}}\)
8. 6 m : 25 cm

9. **ARCHAEOLOGY** It took a team of archaeological volunteers 24 days of steady work to excavate a 30-ft wall. At that rate, how much did the volunteers excavate each day?

10. **WRITING MATH** Which measurement is more precise, 3 in. or 3\(\frac{1}{4}\) in.? Explain.

**Practice Exercises** • For Extra Practice, see page 677.

Complete.

11. 8 qt = \__?\__ c
12. 444 in. = \__?\__ yd \__?\__ ft
13. 2 gal = \__?\__ fl oz
14. 3.2 T = \__?\__ oz
15. 0.4 cm = \__?\__ m
16. 300 mg = \__?\__ g
17. 0.006 kg = \__?\__ g
18. 8.7 mL = \__?\__ L

Name the best customary unit for expressing the measure of each.

19. weight of a TV
20. height of a room
Name the best metric unit for expressing the measure of each.
21. capacity of a reservoir 22. mass of a shovel

Write each ratio in lowest terms.
23. $14 \text{ kg : } 35 \text{ kg}$  24. $80 \text{ m to } 400 \text{ cm}$  25. $\frac{16 \text{ h}}{2 \text{ days}}$

Find each unit rate.
26. $135 \text{ mi in } 3 \text{ h}$  27. $\$15 \text{ for } 250 \text{ copies}$

28. HISTORY An Egyptian merchant ship from 1500 B.C. was about 90 ft long and a Roman galley was about 235 ft long. Write a ratio in lowest terms to express the relationship between lengths of the two ships.

29. Which holds more liquid, a 4-L vase or a 3500-mL vase?

30. Use a centimeter ruler. Measure the width of your desktop to the nearest centimeter. What is the measurement? What is the GPE of the measurement?

31. DATA FILE Use the data on rectangular structures on page 645. What is the GPE for the measurements of the length and width of the Wat Kukut Temple? of the Parthenon?

32. NUMBER SENSE The capacity of a cup is either 0.25 L, 2.5 L, or 25 L. Which measurement makes the most sense?

33. Loch Ness has a capacity of about 2000 billion gal of water. If you were to drain the lake to look for the “monster,” how many quarts of water would you have to remove?

34. The ratio of boys to girls at O’Neal High is 5:6. If there are 308 students, how many are boys and how many are girls?

EXTENDED PRACTICE EXERCISE
35. The 1.5-mi walking trail through the main ruins area at Bandelier National Monument in New Mexico passes by caves, cliff ruins, petroglyphs, and rock carvings from this ancient village. Most people walk the trail in 45 min. What is their walking speed in miles per hour?

36. According to the early Greeks, if the ratio of the length to the width of a rectangle is 1.6:1, it is a Golden Rectangle. Why do you think rectangles with this shape are “golden”? What objects in the classroom or in daily life have nearly this same shape?

MIXED REVIEW EXERCISE
Find the measure of each angle. (Lesson 4-1)
37. $\angle ABC$  \hspace{1cm} \angle BCA \\ \hspace{1cm} \angle CAB$

38. $\angle DEF$  \hspace{1cm} \angle EFD \\ \hspace{1cm} \angle FDE$

mathmatters3.com/self_check_quiz
Work with a partner.

What if the ancient Romans had invented basketball or football? Imagine spectators cheering slam dunks and touchdowns instead of gladiator fights!

The arena within the Roman Colosseum was oval-shaped and had an area of about 40,000 ft². Could a basketball court fit within the arena? Was the arena large enough for a football field? Explain your thinking.

BUILD UNDERSTANDING

When solving a problem involving measuring a plane figure, you may need to decide whether the problem requires finding the distance around the figure, or the amount of surface the figure covers, or both. When you know which measurement you want, apply the correct formula.

Recall that the distance around a polygon is its **perimeter**, the distance around a circle is its **circumference**, and the amount of surface a figure covers is its **area**.

**Example 1**

**ARCHAEOLOGY** What is the width of the fence around an archaeological dig if the region enclosed is a rectangle with a perimeter of 68 m and a length of 24.4 m?

**Solution**

The situation involves perimeter. Use the formula \( P = 2l + 2w \).

\[
P = 2l + 2w
\]

\[
68 = 2(24.4) + 2w \quad \text{Substitute.}
\]

\[
68 = 48.8 + 2w \quad \text{Subtract 48.8 from each side.}
\]

\[
19.2 = 2w \quad \text{Multiply each side by \( \frac{1}{2} \).}
\]

\[
9.6 = w
\]

The fence is 9.6 m wide.

**Reading Math**

The perimeter of a figure means “the measure all around it.” The term comes from two Greek words—peri, meaning “all around,” and metron, “measure.” What other words can you think of that are derived from peri and/or metron? Check your choices with a dictionary.
Example 2

The largest pizza ever made measured 122 ft 8 in. in diameter. If your classmates were to share this pizza equally, about how many square inches of pizza would each get? Use 3.14 for $\pi$.

Solution

First, find the area of the pizza. Use the formula $A = \pi r^2$.

\[
A = \pi r^2 = 3.14 \times 736^2 \quad \text{Use a calculator.}
\]

\[
= 1,700,925.44 \quad \text{Round your answer.}
\]

The pizza had an area of about 1,700,925 in.$^2$. Divide by the number of students in your class to find out how much pizza each gets.

Example 3

Find the area of this figure.

Solution

The figure can be divided into a rectangle and a triangle.

rectangle  triangle  
\[
\begin{align*}
A &= lw \\
\quad &= (8.5)(6) \\
&= 51
\end{align*}
\]

\[
\begin{align*}
A &= \frac{1}{2}bh \\
\quad &= (0.5)(4.5)(6) \\
&= 13.5
\end{align*}
\]

The area of the figure is the sum of the areas of the rectangle and triangle. The area is $51 \text{ m}^2 + 13.5 \text{ m}^2$, or $64.5 \text{ m}^2$.

Example 4

What is the area of the shaded region of this figure? Use 3.14 for $\pi$.

Solution

The shaded area is the difference between the areas of the circles.

\[
\begin{align*}
A &= \pi r^2 \\
\quad &= 3.14(9^2) \\
&= 254.34 \text{ ft}^2
\end{align*}
\]

\[
\begin{align*}
A &= \pi r^2 \\
\quad &= 3.14(5^2) \\
&= 78.5 \text{ ft}^2
\end{align*}
\]

Subtract: $254.34 - 78.5 = 175.84$

The area of the shaded region is about $175.84 \text{ ft}^2$. 

Math:
Who, Where, When

In traditional African societies, as well as in traditional societies elsewhere, the circular house is a common shape. Among the many reasons for this is a geometric one. You can consider the example of the Kikuyu house to see why. The diameter of the base of one of these Kenyan houses is typically about 14 ft and its circumference is about 44 ft. First, find what the area would be for a square with a perimeter of 44 ft, and then for a different rectangle with that same perimeter. What do you notice?
TRY THESE EXERCISES

Find the perimeter or circumference of each. Then find the area of each. If necessary, round answers to the nearest whole number.

1.  
   \begin{array}{c}
   \text{2.6 m} \\
   \end{array}

2.  
   \begin{array}{c}
   5 \text{ ft} \\
   4 \text{ ft} \\
   7 \text{ ft} \\
   \end{array}

3.  
   \begin{array}{c}
   3.5 \text{ cm} \\
   \end{array}

4.  
   \begin{array}{c}
   11 \text{ ft} \\
   \end{array}

PRACTICE EXERCISES • For Extra Practice, see page 677.

5. What is the perimeter of a regular octagon with 5-cm sides?
6. What is the circumference of a circle with a radius of 6.6 m?
7. Find the height of a triangle if area = 24 cm² and base = 10 cm.

Find the area of the shaded region of each figure.

8.  
   \begin{array}{c}
   12 \text{ ft} \\
   5.5 \text{ ft} \\
   \end{array}

9.  
   \begin{array}{c}
   6 \text{ m} \\
   4 \text{ m} \\
   \end{array}

10.  
    \begin{array}{c}
    3.25 \text{ in.} \\
    6.5 \text{ in.} \\
    \end{array}

11. If you triple the length of the radius of a circle, how does the circumference change?
12. DATA FILE Use the data on page 645 to find the information needed to answer this question. Which has the greater area, the base of the Ziggurat of Ur or the Parthenon in Athens? Use mental math.
13. ARCHAEOLOGY A 1000-year-old Anasazi kiva is in the shape of a circle. If the area of the kiva is about 1661 ft², what is the distance around it?
14. STAGE DESIGN A stage from an ancient amphitheatre is shaped like a trapezoid. The front of the stage is 30 ft across, the back is 40 ft across, and the distance from front to back is 25 ft. If a circular region of the stage, 6 ft across, is designated as a pond, what is the area of the space left for actors to walk in?
15. SPORTS The distance from one base to the next in a standard baseball diamond is 90 ft. If the ratio of that length to the length of a basepath in a Little League diamond is 3:2, what is the area of a Little League diamond?
16. WRITING MATH Since π is an irrational number, many calculations involving π are found using the approximations 3.14 or \( \frac{22}{7} \). When might it be easier to use \( \frac{22}{7} \) rather than 3.14 to estimate area or circumference?
17. TALK ABOUT IT Irene and Luis both used calculators to find the area of a circle with a radius measuring 2.5 cm. Irene got 19.63495408 cm² and Luis got 19.625 cm². How can you account for the difference in their answers?
18. The side of a square is equal in length to the diameter of a circle. Which figure will have the greater area, the square or the circle?
RECREATION A community has set aside a rectangular area 250 ft by 200 ft to use for a swim center. The board of directors wants the park to have four pools:

- Pool A—a circular wading pool for small children
- Pool B—a large rectangular lap pool
- Pool C—a smaller L-shaped pool
- Pool D—a pool with an appealing irregular shape that has semi-circular as well as rectangular regions

The park should also have a small circular fountain and a shower area. It may also have picnic tables, chairs, and a food concession.

19. Imagine that you are on the planning board. Design a park layout. Submit a detailed sketch showing the size and location of each pool and feature. (You may want to use graph paper.)

20. Find the area of each pool.

21. Suppose you decide to make the pools safer by placing a border of 1-ft² tiles around each. If each tile costs $5, what will be the total cost of the number of tiles you need?

22. CHAPTER INVESTIGATION Choose a manmade structure, either ancient or modern. Begin research to learn the structure's length, width, and height. Identify any special characteristics. Then make a rough sketch of the structure and label any known measurements.

---

EXTENDED PRACTICE EXERCISES

23. Suppose you were going to paint all the walls of your classroom. What must you know to find how much it will cost and how long it will take?

24. If all sides of the figure at the right are either parallel or perpendicular, then what is the perimeter of the unshaded portion? Does it matter what size the shaded region is?

25. A farmer has 100 ft of fence. Can the farmer enclose more pasture for grazing with a square or a circular enclosure? Is this true for any length of fence?

---

MIXED REVIEW EXERCISES

Can the given measures be the lengths of the sides of a triangle? (Lesson 4-6)

26. 5 cm, 8 cm, 10 cm  
27. 6 in., 9 in., 5 in.
28. 8 ft, 9 ft, 17 ft  
29. 3 m, 9 m, 11 m
30. 14 yd, 18 yd, 36 yd  
31. 7 km, 3 km, 5 km
32. 9 dm, 9 dm, 16 dm  
33. 4 mi, 10 mi, 10 mi
34. 8.6 m, 5.8 m, 15.3 m

Use the number line at the right for Exercises 35–40. Find each length. (Lesson 3-1)

35. \( \overline{MP} \)  
36. \( \overline{QS} \)  
37. \( \overline{PS} \)
38. \( \overline{MR} \)  
39. \( \overline{NR} \)  
40. \( \overline{MS} \)
Complete.
1. 48 c = ______ qt
2. 512 fl oz = ______ gal
3. 11,600 oz = ______ T
4. 7000 mm = ______ cm
5. 7.03 L = ______ mL
6. 48 g = ______ kg

Write each ratio in lowest terms.
7. 120 m : 2700 cm
8. 3 yd : 48 in.
9. 7 days to 120 hours
10. 5 kg to 5,000,000 mg
11. 1200 min : 1 day
12. 1 yd² : 3 ft²

Choose the best estimate for each.
13. length of basketball court
   a. 90 in.
   b. 90 ft
   c. 90 yd
14. weight of an infant
    a. 9 kg
    b. 9 g
    c. 9 mg
15. length of a drinking straw
    a. 20 m
    b. 20 mm
    c. 20 cm

Solve.
16. What is the unit rate for a train which travels 805 mi in 7 h?
17. Which is the better buy, 1 gal of milk for $1.79, or 1 pt of milk for $0.25?
18. The ratio of girls to boys at South High School is 3:4. If there are 980 students, how many are boys?

Find the perimeter or circumference of each.
19. 20. 21.

22–24. Find the area of each figure in Exercises 19–21.

Find the area of the shaded region in each figure.
25. 26. 27.
28. If you triple the length of the radius of a circle, how does the area change? (Lesson 5-2)

Complete. (Lesson 5-1)

29. 6.5 gal = _____ fl oz
30. $6\frac{5}{8}$ kg = _____ g
31. 816 in. = _____ yd _____ ft

32. 2,500,000 mL = _____ kL
33. 0.04 cm = _____ m
34. 56 c = _____ gal _____ qt

Write each ratio in lowest terms. (Lesson 5-1)

35. 1000 m : 10 km
36. 8.5 T to 34,000 lb
37. 75 cL : 25 mL
38. 10 yd to 540 in.
39. 9.2 mL : 9.2 L
40. 1 gal : 1 fl oz

41. Which holds more liquid, a 200 L barrel or a 22,500 mL barrel? (Lesson 5-1)

Find the perimeter or circumference of each figure. Then find the area of each figure. (Lesson 5-2)

42. 

43. 

44. 

Ancient city ruins often lay under many tons of dirt, debris, rock, or even water. Some sites are overgrown with vegetation. To get to these ruins, archaeologists must employ workers who can operate earth-moving machines such as bulldozers, conveyors, trench excavators, hoists, winches, backhoes, and cranes.

To remove trees from the ground area above ancient ruins, the project site has been divided into three sections—a rectangle, a triangle and a half circle.

1. Find the area in square feet of each section.

To clear 10 ft$^2$ of vegetation requires 3 workers paid $12.50/h and 2 machine operators paid $16/h. Working together, these workers can clear 10 ft$^2$ in 2 h.

2. How long will it take the workers to clear each section?

3. What is the cost to clear the triangular section of vegetation?
Play this game in groups of 3–4 students. You will need the bottom of a large 16-in. pizza box, a 4-in. square paper, and a small coin.

1. One person plays against the other members of the group. This person puts the box on a table and tapes the paper square anywhere on the bottom of the box.

2. The remaining members of the group sit on the floor a few feet away (so that the placement of the square cannot be seen). These players take five turns each tossing the counter into the box.

3. A player on the floor wins if the coin comes to rest completely within the paper square once. The person placing the square wins if a counter never lands on the paper square.

4. Is there a way to determine the chance of landing within the paper square? Explain. How can you find the chance of landing elsewhere within the box? Do you think the game is fair? Explain your thinking.

**Check Understanding**

The probability of an event is a number between 0 and 1. What is the probability of an event that will always occur? What is the probability of an impossible event? Give an example of each.

**Example 1**

What is the probability that a point chosen at random from within M is also in N?

**Solution**

Find the probability.

\[ P = \frac{\text{area of } N}{\text{area of } M} \]

\[ = \frac{16}{256} \]

\[ = \frac{1}{16} \text{ or } 0.0625 \]

So the probability that a point chosen at random is in N is \( \frac{1}{16} \), or 0.0625.
Example 2

A dart is dropped onto a foam board shown at the right. What is the probability that the dart lands in the blue region? In the green region?

Solution

\[ P(\text{blue}) = \frac{\text{area of blue circle}}{\text{area of larger circle}} = \frac{(\pi)(3^2)}{(\pi)(6^2)} = \frac{9}{36} = \frac{1}{4} \]

Since \( P(\text{green or blue}) = 1 \), then \( P(\text{green}) = 1 - P(\text{blue}) \). \( P(\text{green}) = \frac{3}{4} \).

Example 3

A treasure chest was buried long ago beneath what is now school property. No one knows where the chest lies. If the school property is a rectangle measuring 600 ft by 540 ft, what is the probability that the chest could be found by excavating the baseball diamond, a square with sides of 90 ft each?

Solution

\[ P(\text{chest in diamond}) = \frac{\text{area of diamond}}{\text{area of property}} \]

\[
\text{area of diamond} = A = s^2 = 90^2 = 8100
\]
\[
\text{area of property} = A = lw = (600)(540) = 324,000
\]

\[ P(\text{chest in diamond}) = \frac{8100}{324,000} = \frac{1}{40} = 0.025 \]

The probability is \( \frac{1}{40} \) or 0.025.

Example 4

GAMES Twenty-five darts are randomly thrown at a circular dartboard and all of the darts land within the dartboard. Four hit the bull's-eye. If the diameter of the bull's-eye is 24 cm, what is the approximate area of the dartboard?

Solution

Since 4 of 25 darts landed in the bull's-eye, the probability of a single dart hitting the bull's-eye is \( \frac{4}{25} \).

\[ P(\text{dart landing in bull's-eye}) = \frac{\text{area of bull's-eye}}{\text{area of dartboard}} = \frac{4}{25} \]

Area of bull's-eye = \( \pi r^2 \approx 3.14 \cdot 12^2 \approx 452.16 \text{ cm}^2 \)

Let \( x = \text{area of dartboard} \). \( \frac{4}{25} \approx \frac{452.16}{x} \); \( x \approx 2826 \)

The area of the dartboard is about 2826 cm\(^2\).
TRY THESE EXERCISES

Find the probability that a point selected at random is in the shaded region.

1. 2. 3. 4. GAMES Refer to the game in the opening situation. What is the probability of landing the coin on the small square if the box measures 12 in. on each side? Disregard the area of the coin.

GAMES A standard deck of playing cards has 52 cards. A card is drawn at random from a shuffled deck. Find each probability.

5. $P($queen$)$ 6. $P($red card$)$ 7. $P($black face card$)$

Find the probability that a point selected at random is in the shaded region.

8. 9. 10. 11. 12. 13. 14. Suppose a cordless phone has been left somewhere within a 2000 ft$^2$ house. What is the probability it is in the 20-ft by 15-ft living room?

15. The total area of the state of Oklahoma is 69,919 mi$^2$. The area of its capital, Oklahoma City, is 604 mi$^2$. If a meteor were to land somewhere in the state, estimate the probability that it would land within the city limits of the capital.

16. ARCHAEOLOGY The rectangular foundation for an ancient building measures 150 ft by 80 ft. The foundation is made from stone cubes, with sides measuring 2 ft each. A scroll is hidden within one of the blocks. What is the probability of finding the scroll if a block is chosen at random?
17. **YOU MAKE THE CALL** A square (side = 2 in.) is placed inside a larger square (side = 6 in.). Evan says that the probability of selecting a point at random within the smaller square is \( \frac{1}{3} \), since the ratio of the smaller square's side to the larger square's side is 2:6, or 1:3. Do you agree with Evan's thinking? Explain your reasoning.

Tell whether each event is certain, likely, unlikely, or impossible.

18. **WEATHER** It will snow in July where you live.

19. You will roll a sum of 5 or greater using two number cubes.

20. You left a pencil in one of 5 classrooms, but you don't know which one. You find it in the first room you search.

21. There is life on other planets.

Find the probability that a point selected at random in each figure is in the shaded region.

22. 23. 24. 25. Suppose a leak occurs from above the room shown at the right. What is the probability the leak will be over the carpet?

26. Draw a figure containing a shaded region, so that the probability is 1 out of 6 that a point selected at random will be in the shaded region.

**EXTENDED PRACTICE EXERCISES**

27. **WRITING MATH** In a scale drawing of the ruins of Pompeii, 1 in. = 400 ft. A student is erasing a pencil mark accidentally made somewhere on the drawing. What additional information do you need to know to find the probability that the mark was made on the Palaestra?

28. Suppose a square target looks like the one at the right. What is the probability of hitting the shaded region?

**MIXED REVIEW EXERCISES**

On a coordinate plane, sketch the triangle with the given coordinates. Then classify the triangle by both its angles and its sides. (Lesson 4-1)

29. \( A(5, 5), B(-4, -4), C(-4, 5) \)  
   30. \( L(3, 1), M(9, -1), N(-2, -4) \)
31. \( R(6, 1), S(-3, -2), T(6, -5) \)  
   32. \( X(-2, -2), Y(-1, 2), Z(6, -3) \)

Given \( f(x) = 5x - 8 \) and \( g(x) = 1.3x - 4.7 \), find each value. (Lesson 2-2)

33. \( f(-5) \)  
   34. \( f(13) \)  
   35. \( f(-8) \)  
   36. \( f(9) \)
37. \( g(12) \)  
   38. \( g(-17) \)  
   39. \( g(34) \)  
   40. \( g(-27) \)

mathmatters3.com/self_check_quiz
Problem Solving Skills:
Irregular Shapes

Sometimes you can find the answer to a difficult problem by breaking it into smaller problems you already know how to solve.

Problem

An architect’s sketch of the plan for one floor of a new archaeological museum is shown. What is the area of the floor?

Solution

Solve a simpler problem. Copy or trace the outline of the floor plan on graph paper. Ignore all the inner walls. Divide the floor into 6 figures: a trapezoid, 4 rectangles, and a half-circle. Label them A–F.

Find the area of each figure. Use 3.14 for $\pi$.

A. $\frac{75 + 45}{2} \times 15 = 900$ ft$^2$

B. $30(195) = 5850$ ft$^2$

C. $75(255) = 19,125$ ft$^2$

D. $30(195) = 5850$ ft$^2$

E. $45(75) = 3375$ ft$^2$

F. $\pi \times 37.5^2 \times 0.5 = 2208$ ft$^2$

Add to find the total area of all the regions. The area of the first floor of the museum is 37,308 ft$^2$.

Check your answer by tracing the outline again and dividing it into a different arrangement of plane figures.

Check Understanding

Why do you multiply by $0.5$ when finding the area of region F?

Try These Exercises

1. CARPETING This layout of a wing at a natural history museum shows the African Peoples, Asian Peoples, and Birds of the World galleries. How much carpeting is needed for this wing?

2. An archaeological team has 30 ft of fencing with which to enclose a rectangular region. If the length and width are whole numbers, what different areas, in square feet, are possible?
3. A display case planned for showing fragments of 3500-year-old frescoes will have the shape at right. If there will be 3 glass shelves, how much glass, in square feet, is needed?

\[
\begin{array}{c}
8 \text{ ft} \\
4.75 \text{ ft} \\
4 \text{ ft} \\
4.75 \text{ ft} \\
8 \text{ ft}
\end{array}
\]

**Practice Exercises**

4. How many small squares are in a checkerboard? How many squares of all sizes are there?

5. **ART** Painters are to cover the entire side of this wall from an abandoned factory with a mural. What is the area of the region to be painted?

\[
\begin{array}{c}
6.5 \text{ m} \\
3 \text{ m} \\
3.25 \text{ m} \\
2 \text{ m} \\
3.25 \text{ m} \\
6.5 \text{ m}
\end{array}
\]

6. **WRITING MATH** Write your own problem involving the area of irregular figures that can be solved by first solving a simpler problem or problems.

7. **RECREATION** The Mayans played a game resembling our game of volleyball on courts that looked like the one shown below. Imagine for a moment that you are the coordinator of a tournament for which you will need a rectangular field containing three side-by-side courts like these. You decide that the courts must be at least 20 m apart and that there must be at least 5 m between an edge of a court and the perimeter of the field. How long must your field be? How wide?

\[
\begin{array}{c}
25 \text{ m} \\
90 \text{ m} \\
15 \text{ m} \\
30 \text{ m} \\
15 \text{ m}
\end{array}
\]

**Mixed Review Exercises**

Use the polygon-sum theorem to find the sum of the measures of convex polygons with the given number of sides. Then find the measure of each interior angle, assuming each polygon is regular. Round to the nearest hundredth if necessary. (Lesson 4-7)

8. 23  
9. 45  
10. 38  
11. 28

12. 18  
13. 34  
14. 50  
15. 42

16. **DATA FILE** Use the data on page 645 on housing units. What percentage of mobile homes or trailers are vacant? (Prerequisite Skill)
PRACTICE ■ Lesson 5-3

Find the probability that a point selected at random is in the shaded region.

1. 

2. 

3. 

4. 

5. 

6. 

7. Suppose a book has been left somewhere within a 2700 ft² house. What is the probability that the book is either in the 15-ft by 10-ft den or a 12-ft by 10-ft bedroom?

Tell whether the event is certain, likely, unlikely, or impossible.

8. The temperature will hit 90 degrees Fahrenheit in December.

9. You will draw a red face card from a standard, shuffled deck of cards.

10. The area of a rectangular region is calculated by multiplying its length by its width.

11. You will leave school grounds before 6 P.M. this evening.

PRACTICE ■ Lesson 5-4

Find the area of each figure.

12. 

13. 

14. 

15. A counter top for a kitchen will have the shape shown. How much laminate material, in square inches, is needed to make this counter top?

16. How many rectangles of all sizes are in the figure shown?
Write each ratio in lowest terms. (Lesson 5-1)

17. 98 m:49 km
18. 13 c to 4 fl oz
19. 1760 yd:10,560 ft

Find the perimeter or circumference of each figure. Then find the area of each figure. Use 3.14 for \( \pi \). (Lesson 5-2)

20.

21.

22.

23. Suppose a parachutist will be landing in the region at the right. What is the probability that she will land in the shaded part of the region? (Lesson 5-3)

24. If everyone in this classroom shakes hands with everybody else exactly once, how many handshakes will occur? (Lesson 5-4)

Mid-Chapter Quiz

Write each ratio in lowest terms. (Lesson 5-1)

1. 25 in. to 5 ft
2. 48 c : 320 oz
3. Which is the better buy, a 19-oz box of cereal for $2.66 or the 12-oz box of the same cereal for $1.92?
4. The diagonal of a rectangle is 25 cm. If the long side of the rectangle is 24 cm, what is the ratio of the short side to the diagonal?

Find the perimeter and area of each figure. Round answers to the nearest tenth. (Lesson 5-2)

5.

6.

Draw the described figures. Find the probability that a randomly chosen point inside the larger figure lies outside the shaded area. (Lesson 5-3)

7. Draw a circle with a radius of 6 cm. Draw a smaller circle inside the first with a radius of 3 cm. Shade the smaller circle.
8. Draw a square with sides measuring 10 cm each. Find the midpoint of one side. Connect the point to one of the endpoints of the opposite side. Shade the right triangle.
Three-Dimensional Figures and Loci

**Goals**
- Analyze space figures.

**Applications**
Archaeology, Architecture, Art

Trace and cut out each pattern below. Try to fold each to form a three-dimensional figure. What do you notice?

---

**BUILD UNDERSTANDING**

A **polyhedron** (plural: **polyhedra**) is a closed, three-dimensional figure in which each surface is a polygon. The surfaces are called **faces**. Two faces intersect at an **edge**, a **vertex** is a point where three or more edges intersect.

A polyhedron with two identical parallel faces is called a **prism**. Each of these faces is called a **base**. Every other face is a parallelogram. A **pyramid** is a polyhedron with only one base. The other faces are triangles that meet at a vertex. A prism is named by the shape of its bases and a pyramid by the shape of its base. The **lateral faces** are those that are not bases. The edges of these faces are called **lateral edges** and can be parallel, intersecting, or skew.

Some three-dimensional figures have flat *and* curved surfaces. A **cylinder** has a curved region and two parallel congruent circular bases. Its **axis** joins the centers of the two bases.
A cone is a three-dimensional figure with a curved surface and one circular base. Its axis is a segment from the vertex to the center of the base.

A sphere is the set of points in space that are the same distance from a given point called the center of the sphere.

Example 1

Identify the figure.

a. b.

Solution

a. Square pyramid—it has one square base and triangular faces.
b. Cone—it has a curved surface and one circular base.

Example 2

For the pentagonal prism at the right, identify the bases, a pair of intersecting faces and the edge at which they intersect, and a pair of skew edges.

Solution

Some answers may vary.

Bases: \(ABCDE\) and \(FGHIJ\)

Pair of intersecting faces: \(AFGB\) and \(BGHC\)

Edge where these two faces intersect: \(\overline{BG}\)

Pair of skew edges: \(CH\) and \(ED\)

Example 3

ARCHAEOLOGY An archaeologist says that a Greek artifact is in the shape of a right hexagonal prism. Draw the prism.

Solution

Step 1: Draw two congruent hexagons on graph paper.

Step 2: Use a straightedge to connect the corresponding vertices. Use dotted lines to show the unseen lateral edges.
Sometimes you will be asked to describe or identify a set of points that meets particular requirements. The mathematical term for specifying points is **locus**, the set of all points that satisfy a given set of conditions. The word *locus* comes from Latin and means *place*; its plural is *loci*.

### Example 4

**Describe the locus of points 6 cm from a given point, P. All points lie within the same plane.**

**Solution**

Draw point P on a sheet of paper. Locate and mark several points 6 cm from it. If you continue to add points to the drawing, what figure is formed? A circle with a radius of 6 cm.

### Try These Exercises

Identify each figure. Then identify the base(s), a pair of parallel edges, intersecting faces, and intersecting edges.

1. 2. 3.

4. Draw a cone.

5. Describe the locus of points equidistant from two parallel lines.

### Practice Exercises • For Extra Practice, see page 678.

Name the polyhedra shown below. Then state the number of faces, vertices, and edges each has.

6. 7. 8.

Draw the figure.

9. **triangular prism**

10. **cylinder**

11. **WRITING MATH** Examine your answers for Exercises 6–8. For each polyhedron, what can you say about how the sum of its faces and vertices compares with the number of edges? Write a rule to describe the relationship among the faces, vertices and edges of a polyhedron.
12. Describe and draw the locus of points in a plane that are 4 m from a given line in the plane.

13. Draw a picture to show the locus of points equidistant from the two sides of \(\angle ABC\) that are in the interior of \(\angle ABC\).

14. Describe the locus of points in space 3 ft from point \(O\). (Hint: A locus of points in space may form a three-dimensional object.)

15. Describe the locus of points in space that are a given positive distance from a given line.

16. **ARCHITECTURE** A 6-story building is 72 ft high. All stories are the same height. Describe the locus of points that are within the building 24 ft from the floor of the fourth floor of the building.

17. **ART** A sculpture is formed by placing an oblique square pyramid on top of a right rectangular prism. The rectangular prism has a square base and its height is twice the length of an edge of the base. The base of the pyramid is the same size as the base of the prism. Draw the sculpture.

18. **CHAPTER INVESTIGATION** Build a three-dimensional model of the structure you have chosen. Break the structure into smaller three-dimensional figures or sections. Then assemble them to make the final product.

---

**EXTENDED PRACTICE EXERCISES**

19. A **cross section** is the two-dimensional figure formed when you cut a three-dimensional shape with a plane. If you cut a cross section of a square pyramid parallel to the base, what polygon will be formed?

20. What would a triangular prism look like if seen from the side? What would it look like from above? Assume that the prism is resting on its base.

---

**MIXED REVIEW EXERCISES**

Find the value of \(x\) in each figure. (Lesson 4-3)

21. \[ \begin{array}{c}
5 \text{ m} \\
5 \text{ m}
\end{array} \]

22. \[ \begin{array}{c}
1.7 \text{ cm} \\
1.7 \text{ cm}
\end{array} \]

23. \[ \begin{array}{c}
2.9 \text{ in.} \\
3.8 \text{ in.}
\end{array} \]

Solve each equation. (Lesson 2-5)

24. \[ 3(2x + 1) - 8 = 5x + 2(x + 1) \]
25. \[ -3x + 4(x - 1) = 6 - 4(x + 2) \]
26. \[ -2(x - 3) + 5 = -5x + 9 \]
27. \[ 3(x - 3) + 2x = 7x - 3(x + 1) \]
28. \[ 5 - 4(x - 8) = -5(4x - 1) \]
29. \[ -4(x - 2) + 3 = x + 2(x - 5) \]
30. \[ 2 - 3(2x - 6) + x = 3x - 5 \]
31. \[ 2(x + 4) - 3x + 8 = -3(x - 4) + 5x \]
32. \[ 4(x + 2) - 3x + 11 = 2(3x - 2) + 3(x + 4) \]
33. \[ -5(2x + 1) + 2(x - 3) = 2x + 6(x + 3) \]
Work with a partner.

Construct a square pyramid out of construction paper, using only the following: straightedge, compass, scissors, and tape. Write a description of how you did it.

BUILD UNDERSTANDING

When you are asked to find the surface area of a three-dimensional figure, think about whether the figure is a prism, pyramid, cylinder, cone, or sphere, or whether it is a combination of figures. To help you identify the shape of each surface, think about what the figure would look like if it were cut apart. Notice whether any surfaces are congruent.

Example 1

PACKAGING A box of Teen Chow cereal is 11.5 in. high, 7.5 in. wide, and 2.5 in. deep. What is the surface area of the box?

Solution

The cereal box is a rectangular prism, so it has 3 pairs of congruent rectangular faces. To find its surface area, find the area of each face. Use the formula \( A = lw \).

\[
\text{SA} = 2(\text{area of front}) + 2(\text{area of side}) + 2(\text{area of top})
\]
\[
= 2(11.5 \times 7.5) + 2(11.5 \times 2.5) + 2(7.5 \times 2.5)
\]
\[
= 172.5 + 57.5 + 37.5
\]
\[
= 267.5
\]
The surface area is 267.5 in.\(^2\).

Example 2

MANUFACTURING At Farrow’s Ceramic Factory ceramic replicas of the Great Pyramid at Giza are made. Each model has a square base 10 cm in length, and triangular faces each with a height of 12 cm. Farrow’s plans to paint the models. What is the surface area of each?
Solution

The model is a square pyramid. Each of the four triangular faces has the same area. To find its surface area, find the area of each face and of the base.

\[
SA = 4(\text{area of triangular face}) + \text{area of square base}
\]

\[
\begin{align*}
\text{area of triangular face} & \quad \text{area of square base} \\
A &= \frac{1}{2}bh & A &= s^2 \\
&= \left(\frac{1}{2}\right)(10)(12) & &= 10^2 \\
&= 60 & &= 100 \\
SA &= 4(60) + 100 = 340
\end{align*}
\]

The surface area is 340 cm².

Example 3

A can of bread crumbs is 14 cm high and 8 cm across. What is the surface area of the can?

Solution

The can is a cylinder. To find its surface area, add the area of the curved surface to the area of the two bases.

\[
\begin{align*}
\text{area of the curved surface} & \quad \text{area of each circular base} \\
A &= 2\pi rh & A &= \pi r^2 \\
&= (2)(\pi)(4)(14) & &= (\pi)(16) \\
&= 351.68 & \approx 50.24 \\
\end{align*}
\]

The can has two congruent circular bases. \((2)(50.24) = 100.48\)

\[
SA \approx 351.68 + 100.48 \approx 452.16
\]

The surface area of the can is approximately 452.16 cm².

Example 4

A tent in the shape of a tepee is 4 m across with a slant height of 2.6 m. What is the surface area of the canvas, including the floor?

Solution

The tent approximates a cone. To find its surface area, add the area of the curved surface to the area of the base.

\[
\begin{align*}
SA &= \pi rs + \pi r^2 \\
&= \pi (2)(2.6) + \pi(2)^2 \\
&\approx 16.328 + 12.56 \\
&\approx 29
\end{align*}
\]

The surface area of the tent is approximately 29 m².
Example 5

**SPORTS** What is the surface area of a soccer ball with a diameter of about 9 in.?

**Solution**

The soccer ball is a sphere. To find its surface area, use the formula \( SA = 4\pi r^2 \).

\[
SA = (4)(3.14)(4.5)^2 \\
= 254
\]

The soccer ball has a surface area of about 254 in.²

---

### Try These Exercises

Find the surface area of each figure. Assume that the pyramid is a regular pyramid. Use 3.14 for \( \pi \).

1. 2. 3.

4. **DATA FILE** Use the data on the sizes and weights of various balls used in sports on page 653. Calculate the surface area of a volleyball.

---

### Practice Exercises • For Extra Practice, see page 679.

Find the surface area of each figure. Assume that all pyramids are regular pyramids. Use 3.14 for \( \pi \). Round answers to the nearest whole number.

5. 6. 7. 8.

9. What is the surface area of a square pyramid whose base length is 8 m and whose faces have heights of 6.4 m?

10. **ARCHITECTURE** The Marina Towers in Chicago are cylindrical shaped buildings that are 586 ft tall. There is a 35-ft diameter cylindrical core in the center of each tower. If the core extends 40 ft above the roof of the tower, find the exposed surface area of the core.
Find the surface area of each figure. Use 3.14 for π. Round answers to the nearest whole number.


14. Use mental math.
Which has the greater surface area, the can or the box?

15. **ART** The base of a sculpture is a regular pentagonal prism with sides 10 cm high and 6 cm wide. What additional information do you need to find the surface area of the base of the sculpture?

16. **WRITING MATH** The surface area of a rectangular prism is 178 in.². What is the height of the figure if its length is 3 in. and its width is 4 in.? Explain how you got your answer.

17. **SPORTS** The “shots” that shot-putters toss are heavy spheres that range in diameter from 95 mm to 130 mm. What is the difference in surface area between the largest and smallest shot?

18. **ASTRONOMY** Jupiter, the largest planet, has an equator with a diameter of about 88,000 mi. To the nearest million miles, what is the surface area of Jupiter? Assume that it is a sphere.

---

### Extended Practice Exercises

19. What happens to the surface area of a cube if you (a) double the length of a side, or (b) divide the length of a side by 3?

20. One way to express the formula for finding the surface area of a cylinder is \( SA = 2\pi rh + 2\pi r^2 \). How else can this be expressed?

21. To paint the sides of a cube, 1 quart of paint is used. Suppose two such cubes are glued together to form a rectangular solid. How much paint will it take to paint the new rectangular solid?

22. **CHAPTER INVESTIGATION** Estimate the surface area of the ancient structure you have chosen. Explain how you got your answer.

---

### Mixed Review Exercises

Find each length. (Lesson 3-1)

23. In the figure below, \( AC = 106 \).
Find \( AB \).

24. In the figure below, \( RT = 170 \).
Find \( ST \).
Review and Practice Your Skills

Practice Lesson 5-5

Name each three-dimensional figure shown below. Then state the number of faces, vertices, and edges for each.

1. 2. 3.

4–6. For the figures in Exercises 1–3, identify the following:
   a. base(s)
   b. a pair of parallel edges
   c. a pair of intersecting faces
   d. a pair of intersecting edges

Draw each figure.

7. oblique cone
8. pentagonal prism
9. oblique hexagonal pyramid

10. Describe and draw the locus of points that are inside or on a square and equidistant from two adjacent sides of the square.

11. Describe and draw the locus of points in a plane equidistant from a line and a point that is not on the line.

Practice Lesson 5-6

Find the surface area of each figure. Assume the pyramid is a regular pyramid. Round answers to the nearest tenth.


Find the surface area of each figure. Round answers to the nearest tenth.

15. 16. 17.

18. Find the surface area of a square pyramid with base length = 10 m and faces with heights of 8.2 m.
19. What happens to the surface area of a sphere if you triple the radius? (Lesson 5-6)
20. What happens to the surface area of a cone if you double the radius? (Lesson 5-6)

Find each unit rate. (Lesson 5-1)
21. 260 mi in 4 h
22. $42 for 1400 stamps
23. 51 gal in 1.5 min

Find the probability that a point selected at random in each figure is in the shaded region. (Lesson 5-3)
24. 
25. 
26. 

Find the surface area of each figure. Assume the pyramid is a regular pyramid. Round answers to the nearest whole number. (Lesson 5-6)
27. 
28. 

Archaeologists trace the histories of ancient civilizations by studying ancient records and artifacts. Many hours are spent in the field searching for buried cities and artifacts. Archaeologists make detailed maps of each site they discover. They analyze the layout of buildings and rooms. Each item found is labeled and catalogued. Archaeologists note the exact location artifacts are found in order to determine their purpose.

Suppose you are excavating a building buried under several feet of ancient volcanic ash and silt. By reading inscriptions on stones, you expect to find a cylindrical pedestal, 3 ft in diameter, somewhere in the interior of the room.

1. If the building is circular with a diameter of 25 feet, what is the probability of finding the pedestal if a dig site is chosen randomly?
2. The site can be divided into two rectangular sections: the first measuring 15 ft by 13 ft, and the second, 20 ft by 20 ft. Find the total area of the site.
Work in groups of three or four students.

Many environmental groups criticize manufacturers for over-packaging their products. On the other hand, over-packaging is one way to make customers think they are getting more for their money.

1. Choose a product that you think uses too much packaging.
2. Develop a new way to package the product that uses less packaging material. Remember, the packaging must keep the product from breaking, fit neatly in shipping cartons, and look appealing to the consumer.
3. Make a packaging sample for display.

**Build Understanding**

Recall that volume is a measure of the number of cubic units needed to fill a region of space. To find the volume of a three-dimensional figure, first you must determine whether the figure is a prism, pyramid, cylinder, cone, sphere, or a combination of shapes. Then apply the appropriate formula or formulas for volume.

**Example 1**

Find the volume of the figure at the right.

**Solution**

The figure is a prism. To find the volume \( V \) of any prism, multiply the area of the base \( B \) by the height \( h \) of the prism. First find the area of the base, which is a right triangle.

\[
B = \frac{1}{2}bh
\]

\[
= \left(\frac{1}{2}\right)(8)(6)
\]

\[
= 24
\]

The area of the base is 24 in.\(^2\). Then use the volume formula.

\[
V = Bh
\]

\[
= (24)(12)
\]

\[
= 288
\]

The volume is 288 in.\(^3\).
A three-dimensional figure may be a combination of shapes. Mentally break the figure into smaller pieces. Then find the volume of each piece. Finally, use the information to solve the problem.

**Example 2**

Find the volume of the shaded part of the figure shown.

**Solution**

To find the volume of the shaded part, find the difference between the volume of the small pyramid and the volume of the large pyramid.

To find the volume of any pyramid, multiply \( \frac{1}{3} \) of the area of its base \((B)\) by its height \((h)\).

The base of each of these pyramids is a square.

Find the volume of the small pyramid. Find the volume of the large pyramid.

\[
V = \frac{1}{3} Bh \\
V = \frac{1}{3} (9^2)(15) \\
V = 405
\]

The volume of the large pyramid is 3240 cm\(^3\). The volume of the small pyramid is 405 cm\(^3\).

\[3240 - 405 = 2835\]

The volume of the shaded portion is 2835 cm\(^3\).

**Example 3**

**MANUFACTURING** A candy company decides to sell its new Blast Off candy bars in a package shaped like a rocket. The body of the rocket is shown at the right. Find the volume of the figure.

**Solution**

A cylinder and cone combine to form the figure shown. Add the volume of the cone to the volume of the cylinder.

Find the volume of the cylinder. Find the volume of the cone.

\[
V = \pi r^2 h \\
V = \frac{1}{3} \pi r^2 h \\
V = (\pi)(3^2)(12) \\
V = \frac{1}{3} (\pi)(3^2)(6) \\
V \approx 339 \\
V \approx 57
\]

The volume of the cylinder is about 339 in.\(^3\). The volume of the cone is about 57 in.\(^3\).

\[339 + 57 = 396\]

The volume of the figure is about 396 in.\(^3\).
Example 4

ASTRONOMY The only asteroid visible to the naked eye is 4 Vesta, discovered in 1807. Its diameter is 323 mi. What is its volume? Assume that 4 Vesta is a sphere.

Solution

To find the volume of a sphere, use the formula \( V = \frac{4}{3} \pi r^3 \).

\[
V = \frac{4}{3} \pi r^3
\approx \left(\frac{4}{3}\right)(3.14)(161.5^3) \approx 17,635,426
\]

The volume of 4 Vesta is approximately 17,635,426 mi³.

Try These Exercises

Find the volume of each figure. Use 3.14 for \( \pi \). Round answers to the nearest whole number.

1. 
2. 
3. 
4. 

5. A prism has a hexagonal base with an area of 24 cm². If the volume of the figure is 144 cm³, what is its height?

Practice Exercises • For Extra Practice, see page 679.

Find the volume to the nearest whole number. Use 3.14 for \( \pi \).

6. 
7. 
8. 
9. 

10. ARCHAEOLOGY One room in a 12th-century cliff dwelling is in the shape of a rectangular prism. The floor measures 10 ft by 12.5 ft, and the ceiling is 7 ft high. What is the volume of the room?

11. How many cubic meters of water can a water tank hold if the tank is a cylinder 9 m high and 6 m in diameter?

12. A pyramid with a volume of 384 cm³ has a base of area 64 cm². What is the height of the figure?

13. WRITING MATH A pentagonal prism and a pentagonal pyramid prism have congruent bases and the same height. Describe the relationship between the volumes of the two figures.
Find the volume of each. Use 3.14 for \( \pi \). Round answers to the nearest whole number.

14. [3 mm, 5 mm, 8 mm]

15. [4 yd, 4 yd, 24 yd]

16. [3 cm, 6 cm, 9 cm]

17. **ARCHAEOLOGY** The height of the pyramid at Cheops is greater than the height of the Quetzalcoatl pyramid at Cholula, but its volume is less. How can that be?

18. The base of a rectangular pyramid has sides of 3.14 m and 12 m. A cone has the same height and volume. What is its radius? **Hint:** Write and solve an equation; use 3.14 for \( \pi \).

19. What happens to the volume of a cone if its height is tripled?

20. If the radius of a cylinder is tripled, what happens to its volume?

21. Describe how you could find the volume of a crystal belonging to the tetragonal system.

22. **CHAPTER INVESTIGATION** Estimate the volume of your structure using your understanding of three-dimensional figures. Using an index card, make a fact list about your structure. Include the date the structure was built, the name of the architect, materials used, dimensions, surface area and volume. Display your model and fact list.

### Extended Practice Exercises

23. **COST ANALYSIS** A can of Iguana Goodies that is 14 cm high with a radius of 6 cm sells for $1.70. Another can is the same height, but with a radius of 3 cm. It sells for $0.40. Which can of Iguana Goodies is the better buy?

24. A rectangular prism is 12 ft long and 3 ft wide. If its volume is 288 ft\(^3\), what is its surface area?

25. **PACKAGING** Suppose you have a square sheet of cardboard 24 cm on a side. You want to cut equal squares from the corners of the large square to fold the cardboard into an open box. If you cut corners of 1 cm, 2 cm, 3 cm and so on, which size cuts will give you the box with the greatest volume?

### Mixed Review Exercises

Find the perimeter or circumference and the area of each. Use 3.14 for \( \pi \). Round answers to the nearest hundredth if necessary. (Lesson 5-2)

26. [8.7 m, 2.4 m]

27. [12 ft, 8 ft]

28. [5.8 cm]

29. [6.1 in., 3.1 in.]

[mathmatters3.com/self_check_quiz](http://mathmatters3.com/self_check_quiz)
Chapter 5 Review

VOCABULARY

Choose the word from the list that best completes each statement.

1. Two faces of a polyhedron intersect along a(n) ___?__.
2. A(n) ___?__ is used for measuring distances and drawing curved lines and circles.
3. A ratio that compares two quantities that use different units is a(n) ___?__.
4. A polyhedron with one base is called a(n) ___?__.
5. A three-dimensional figure with a curved surface and one circular base is called a(n) ___?__.
6. Half the smallest unit used to make a measurement is the ___?__.
7. The three-dimensional figure where all the points on the surface are the same distance from the center is called a(n) ___?__.
8. A(n) ___?__ has two bases and lateral faces that are parallelograms.
9. The measure of the amount of space enclosed by a three-dimensional figure is its ___?__.
10. The distance around a polygon is its ___?__.

LESSON 5-1  ■  Ratios and Units of Measure, p. 202

A ratio compares two numbers by division and can be written in three ways. For example: 4 : 5, \( \frac{4}{5} \), or 4 to 5.

For both customary and metric measures, multiply to change from a larger unit of measure to an equivalent measure in a smaller unit. Divide to convert a smaller unit of measure to an equivalent measure in a larger unit.

11. Write 8:20 in lowest terms in three ways.
12. How many grams are in 3 kg?
13. What is the greatest possible error (GPE) of a measurement of 2.46 m?

LESSON 5-2  ■  Perimeter, Circumference and Area, p. 206

To solve a problem involving the distance around a plane figure or the surface covered by it, you must choose the correct formula.

Find the perimeter or circumference of each. Then find the area of each. If necessary, round answers to the nearest whole number.

14. 15. 16.
LESSON 5-3  Probability and Area, p. 212

To determine the probability that a randomly chosen point in a figure will be within a given region inside the figure, use the ratio of the area of given region to the total area of figure.

Find the probability that a point selected at random is in the shaded region.

17. 18. 19.

20. An ice skater skates in a rectangular rink 40 m by 125.6 m. If she were to fall at random inside the rink, what is the probability she will fall within a given circle of diameter 8 m?

LESSON 5-4  Problem Solving Skills: Irregular Shapes, p. 216

To find the total area of an irregular figure, first break it down into smaller shapes whose areas you can add.

Find the area of each figure.

21. 22. 23.

24. A wing of a new house has the shape shown at the right. What is the area of the wing?

LESSON 5-5  Three-dimensional Figures and Loci, p. 220

A polyhedron is a three-dimensional figure in which each face is a polygon.

Identify each figure. Then state the number of faces, vertices, and edges for each figure.

25. 26. 27.

Draw each figure.

28. oblique cylinder  29. right cone  30. rectangular prism
LESSON 5-6  
Surface Area of Three-dimensional Figures, p. 224

To find the surface area of a three-dimensional figure, choose the correct formula for the area of each surface of the figure.

Find the surface area of each figure to the nearest tenth.

31. 32. 33.

34. A cube has a surface area of 294 cm\(^2\). What is the length of a side?

35. A can of corn is in the shape of a cylinder. The diameter of the base is 8 cm and the height of the can is 9 cm. If the curved surface of the can is to be covered with a label, what is the area of the label?

36. Each side of a cube is 5 m long. The height of a cylinder is 5 m and the base of the cylinder has a diameter of 5 m. Which figure has the greatest surface area?

LESSON 5-7  
Volume of Three-dimensional Figures, p. 230

To find the volume of a three-dimensional figure, use the correct formula or formulas.

Find the volume of each figure to the nearest tenth.

37. 38. 39.

40. How many cubic feet of concrete will be needed for a driveway that is 40 ft long, 18 ft wide, and 4 in. deep?

41. A rectangular prism and a rectangular pyramid have congruent bases. The height of the prism is 12 in. If both the prism and pyramid have the same volume, what is the height of the pyramid?

42. How many cubic centimeters are in a cubic meter?

CHAPTER INVESTIGATION

EXTENSION  Write a report about the structure you chose. Be sure to include the measurements and known features of the structure. Explain why you chose the scale you did for your model. State your estimate of the exterior surface area and volume of your structure, and explain what these estimates tell you about the structure.
Chapter 5 Assessment

Write each ratio in lowest terms.
1. 4 yd to 16 ft  
2. 400 m to 2 km  
3. 6 qt to 9 gal  
4. What is the height of a triangle with an area of 40 cm\(^2\) and a base of 12 cm?  
5. How many faces, vertices, and edges does a hexagonal prism have?

For Exercises 6–9, use 3.14 for \(\pi\). Round your answers to the nearest tenth.
Find the area of the shaded region of each. Find the surface area of each.

\[
\begin{align*}
6. & \quad \text{Area of shaded region} = 3.6 \text{ cm} \\
7. & \quad \text{Area of shaded region} = 2.5 \text{ m} \\
8. & \quad \text{Surface area} = 45 \text{ in.} \\
9. & \quad \text{Surface area} = 6.3 \text{ ft}
\end{align*}
\]

Find the probability that a point selected at random in each figure is in the shaded region.

\[
\begin{align*}
10. & \quad \text{Probability} = 3.2 \text{ m} \\
11. & \quad \text{Probability} = 6.5 \text{ yd} \\
12. & \quad \text{Probability} = 19 \text{ m}
\end{align*}
\]

Find the volume of each figure to the nearest tenth. Use 3.14 for \(\pi\).

\[
\begin{align*}
13. & \quad \text{Volume} = 24 \text{ cm}^2 \\
14. & \quad \text{Volume} = 12 \text{ m} \\
15. & \quad \text{Volume} = 9.6 \text{ ft}
\end{align*}
\]

16. You want to paint the figure shown. Which formula will you use to find how much paint you will need?

17. How many 1-ft\(^2\) floor tiles are needed to cover the floor shown below?

18. Which measurement is more precise, 4.003 L or 4.99 L?

19. What is the GPE of the measurement 100.9 cm?

20. A micrometer setting shows the measurement 24.46 mm. What are the upper and lower limits for the measurement if the tolerance is ±0.03 mm?

21. A square board with each side measuring 30 cm has a circle in the center with a diameter of 12 cm. If a dart is thrown at the board, what is the probability that it will land in the center circle? Use 3.14 for \(\pi\). Round to the nearest hundredth.
7. In the figure below, \(ZW\) and \(KN\) are intersecting lines, \(KR \cong RN\), and \(ZR \cong RW\). Which postulate could you use to prove \(\triangle KRZ\) is congruent to \(\triangle NRW\)? (Lesson 4-2)

- A. Angle-Angle-Side Postulate
- B. Angle-Side-Angle Postulate
- C. Side-Angle-Side Postulate
- D. Side-Side-Side Postulate

8. Which quadrilateral has exactly one pair of parallel sides? (Lesson 4-9)

- A. parallelogram
- B. rhombus
- C. square
- D. trapezoid

9. Express the ratio 16 in. to 3 ft as a fraction in simplest form. (Lesson 5-1)

- A. \(\frac{3}{16}\)
- B. \(\frac{4}{9}\)
- C. \(\frac{8}{15}\)
- D. \(\frac{16}{9}\)

10. The perimeter of the rectangle is 42 ft. What is the area of the rectangle? (Lesson 5-2)

- A. 7 ft\(^2\)
- B. 14 ft\(^2\)
- C. 98 ft\(^2\)
- D. 196 ft\(^2\)

Test-Taking Tip

**Question 10**
Sometimes more than one step is required to find the answer. In Question 10, you need to find the value of \(x\). Then you can use the value of \(x\) to determine the length and width of the rectangle. Finally, you can multiply the length times the width to find the area.
Part 2  Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

11. Let $A = \{1, 3, 5, 7, 9\}$ and $B = \{4, 5, 6, 7\}$. What is the least member in $A \cup B$? (Lesson 1-3)

12. Cole had $\frac{7}{8}$ of a tank of gas in the lawn mower. After mowing the grass, he had $\frac{1}{4}$ of a tank. What fraction of a tank did Cole use mowing the lawn? (Lesson 1-4)

13. The Mariana Trench is the deepest part of the Pacific Ocean with a depth of about 36,000 ft. Write this depth in scientific notation. (Lesson 1-8)

14. If $g(x) = |x - 10|$, find $g(7)$. (Lesson 2-3)

15. If $8.1d + 2.3 = 5.1d - 3.1$, what is the value of $d + 10$. (Lesson 2-5)

Ten friends recorded the number of minutes they needed to complete their math homework. Use their data below to answer Questions 16–17.

16, 23, 21, 17, 17, 20, 22, 17, 14, 23

16. What is the mean number of minutes the friends spent on their math homework? (Lesson 2-7)

17. What is the median number of minutes the friends spent on their math homework? (Lesson 2-7)

18. What is the measure of $\angle RME$? (Lesson 3-4)

19. What is the value of $x$? (Lesson 4-1)

20. What is the probability that a point selected at random is in the shaded area? (Lesson 5-3)

21. The sides and bottom of a cylindrical swimming pool with a diameter of 25 ft and height of 4 ft are lined with vinyl. Find the area of the vinyl to the nearest square foot. (Lesson 5-6)

Part 3  Extended Response

Record your answers on a sheet of paper. Show your work.

22. A prism with a triangular base has 9 edges. A prism with a rectangular base has 12 edges. Explain in words or symbols how to determine the number of edges for a prism with a 9-sided base. Be sure to include the number of edges in your explanation. (Lesson 5-5)

23. The diagrams show the design of the trashcans in the school cafeteria.

a. The top and sides of the cans need to be painted. Find the area that needs to be painted. Show your work. (Lesson 5-6)

b. Find the volume of the trash can. Show your work. (Lesson 5-7)

Preparring for Standardized Tests

For test-taking strategies and more practice, see pages 709–724.