Think back through your day and mentally make a list of all the products that you have used. Every one of those products was designed, tested, and built by a manufacturer.

Once a new product is designed and thoroughly tested, engineers break the assembly of the product into steps. Each step will be performed by workers on an assembly line. The engineers, technicians, and line workers use math to ensure that each product is manufactured according to the design specifications.

- **Precision Assemblers** (page 253) work in factories to produce complex goods and must know how to use specialized measuring instruments. They read engineering diagrams called *schematics*.

- **Engineering Technicians** (page 273) design and develop new products to meet all required safety standards. Engineering technicians test designs for product quality and look for ways to keep costs down to make new products affordable to the consumer.
U.S. Goods–Imports and Exports, 2004 (millions of dollars)

<table>
<thead>
<tr>
<th>Category</th>
<th>Exports</th>
<th>Imports</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aircraft</td>
<td>7995</td>
<td>3245</td>
</tr>
<tr>
<td>Apparel</td>
<td>1846</td>
<td>13,037</td>
</tr>
<tr>
<td>Jewelry</td>
<td>1329</td>
<td>3486</td>
</tr>
<tr>
<td>Vegetables</td>
<td>1072</td>
<td>1803</td>
</tr>
<tr>
<td>Glassware, Chinaware</td>
<td>128</td>
<td>753</td>
</tr>
<tr>
<td>Rugs</td>
<td>258</td>
<td>590</td>
</tr>
<tr>
<td>TVs, VCRs</td>
<td>1164</td>
<td>9899</td>
</tr>
<tr>
<td>Synthetic fabrics</td>
<td>1833</td>
<td>1495</td>
</tr>
<tr>
<td>Vehicles</td>
<td>28,042</td>
<td>74,507</td>
</tr>
<tr>
<td>Toys and games</td>
<td>2121</td>
<td>7656</td>
</tr>
<tr>
<td>Computers</td>
<td>2944</td>
<td>7311</td>
</tr>
<tr>
<td>Printed Materials</td>
<td>1423</td>
<td>1219</td>
</tr>
</tbody>
</table>

Data Activity: U.S. Goods–Imports and Exports

Use the table for Questions 1–4.

1. What is the dollar value of the apparel exports?

2. Much of the toys and games sold in the United States is manufactured in other countries. How much greater is the value of imports than exports?

3. Of total exports and imports of computers, what percent is imported? Round to the nearest percent.

4. For which category are exports and imports most nearly balanced?

CHAPTER INVESTIGATION

Manufactured items go through an extensive design and development process. Many companies use focus groups made up of panels of consumers to find new ideas for product improvement. Before a new product is sold in stores, it is tested by selected consumers.

Working Together

Choose a simple product that you use frequently. Gather a focus group of four to five students and brainstorm ideas for product improvement. Finally, draw a model or prototype of the new product and determine how the improvements will add to the cost of the product. Use the Chapter Investigation icons to guide your group.
The skills on these two pages are ones you have already learned. Read the examples and complete the exercises. For additional practice on these and more prerequisite skills, see pages 654-661.

**Graphing Linear Equations**

In this chapter you will learn more about graphing equations on the coordinate plane. It will be helpful to review some of these basic skills.

**Example** Graph the equation \(3x - y = -5\).

---

**Solution**

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>-1</td>
<td>2</td>
</tr>
</tbody>
</table>

Graph each equation on the coordinate plane.

1. \(y = 5x - 3\)
2. \(3x + 8 = 4y\)
3. \(-4x - 3y = 4\)
4. \(3y - x = 7\)
5. \(-3x + 2y = -7\)
6. \(y = -4x - 3\)
7. \(3y = -5x + 7\)
8. \(\frac{1}{2}(x - 3) = 2y\)

**Solving Equations**

In this chapter you will be working with various forms of equations. Remember to always apply the order of operations.

**Solve each equation.**

10. \(5a - 3 = 12\)
11. \(-2a + 4 = 4a - 2\)
12. \(3(a - 2) = -3a + 12\)
13. \(-4(a - 2) = 16\)
14. \(12 - 3a = 2a + 2\)
15. \(-2(a + 4) = 16\)
16. \(\frac{1}{2}(a + 3) - 4 = -2\)
17. \(8 - 2a + 7 = 5\)
18. \(4a - 6 - a = 2\)
19. \(3(a + 3) - 2 = 4(a - 6) + a\)
20. \(2(a - 4) + 6 = 3(a + 2) - 4\)
21. \(a + 2(3a - 6) = 30 - 2(a + 3)\)
22. \(-4(a - 2) + 8 = -2a + 3(a - 2)\)
Solving Compound Inequalities

Example Using the replacement set of the real numbers, graph the solution set for the compound inequality \( x \leq 3 \) and \( x > -5 \).

Graph each inequality: \( x \leq 3 \)

\[
\begin{align*}
\text{Graph the intersection:} & \\
& \text{Graph the solution set for each compound inequality on a number line. Use the replacement set of the real numbers.}
\end{align*}
\]

23. \( x > 3 \) and \( x < 9 \)

24. \( x \leq -1 \) and \( x > -8 \)

25. \( x > -2 \) and \( x \geq 1 \)

26. \( x \leq 5 \) and \( x \geq -3 \)

27. \( x < 4 \) and \( x \leq 2 \)

28. \( x > 5 \) and \( x < -2 \)

29. \( x \geq -4 \) and \( x < -3 \)

30. \( x \geq -2 \) and \( x < 5 \)

31. \( x < 4 \) and \( x > 1 \)

Ordering Rational Numbers

To compare rational numbers that are in mixed formats, you need to convert all values to the same format. Decimal form is often the preferred way to compare and order numbers.

Write the rational numbers in order from least to greatest value.

32. \( 1.156, \frac{1}{8}, \frac{7}{6}, \sqrt{2}, -0.2 \)

33. \( 0.7856, \frac{15}{16}, \frac{5}{7}, \sqrt{0.49}, -1, 1 \)

34. \( -6.42, -6\frac{5}{8}, -0.6, \frac{32}{5}, -6.042 \)

35. \( 0.0025, 0.14500, 0.19, \frac{1}{7}, \sqrt{\frac{4}{5}} \)

36. \( 0.03, \frac{2}{3}, \frac{3}{10}, \sqrt{6}, \frac{5}{6} \)

37. \( 0.05, \frac{1}{24}, \sqrt{2}, \frac{0.049}{15}, \frac{3}{26} \)

38. \( -\frac{3}{11}, \frac{1}{3}, -0.31, \frac{\sqrt{2}}{3}, -3.1 \)

39. \( -2.9, -2.95, -2\frac{9}{11}, -2\frac{13}{14} \)
Work with a partner.

Make identical line segments on geoboards. Lengthen or shorten one of the line segments. Make a right triangle on each geoboard using the line segment as the hypotenuse. Write the ratio of the length of the vertical side to the length of the horizontal side for each triangle. How do the ratios compare?

**BUILD UNDERSTANDING**

Many examples of slope exist in everyday life. We measure the slope, or steepness, of roads, stairs and ramps. In mathematics, an important aspect of a line is its slope. We measure slope as a ratio of the vertical change (rise) to the horizontal change (run).

\[
\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{vertical change (change in y-coordinates)}}{\text{horizontal change (change in x-coordinates)}}
\]

You can find the slope of a line on a coordinate plane by finding the units of change between the coordinates of any two points on the line. The vertical change, or change in \(y\), is found by determining the difference of the \(y\)-coordinates. Likewise, the horizontal change, or change in \(x\), is found by determining the difference in corresponding \(x\)-coordinates of the same two points.

The slope of a line segment containing the point \(A(x_1, y_1)\) and \(B(x_2, y_2)\) is

\[
m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{or} \quad \frac{y_1 - y_2}{x_1 - x_2}.
\]

**Example 1**

Find the slope of \(AB\) containing points \(A(-1, 2)\) and \(B(3, -4)\).

**Solution**

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 2}{3 - (-1)} = \frac{-6}{4} = -\frac{3}{2}
\]

Example 1 shows that a line with a **negative** slope falls from left to right. As the value of \(x\) increases, the value of \(y\) decreases. A line with a **positive** slope rises from left to right. As the value of \(x\) increases, the value of \(y\) increases.
If you know one point on a line and the slope of the line, you can also graph the line.

**Example 2**

Graph the line that passes through \(G(1, 1)\) and has a slope of \(-\frac{3}{4}\).

**Solution**

First, plot the point \(G(1, 1)\). The slope is \(-\frac{3}{4}\), so from \(G\), go down 3 units (because the rise is \(-3\)) and right 4 units (because the run is 4). The point is \((1 + 4, 1 - 3)\) or \((5, -2)\).

Draw a line through \((1, 1)\) and \((5, -2)\).

In a linear equation, you can find the \(x\)-intercept, or \(x\)-coordinate of the point at which the line crosses the \(x\)-axis, by substituting 0 for \(y\) and solving for \(x\). You can find the \(y\)-intercept, or \(y\)-coordinate of the point at which the line crosses the \(y\)-axis, by substituting 0 for \(x\) and solving for \(y\). You can then draw a line through these two points to graph the equation. Writing a linear equation in slope-intercept form is another way to easily find the slope and the \(y\)-intercept of the line. In the slope-intercept form, \(y = mx + b\), \(m\) represents the slope of the line and \(b\) represents the \(y\)-intercept.

**Example 3**

Find the slope and \(y\)-intercept for the line with the equation \(5x + 6y = 6\).

**Solution**

\[
5x + 6y = 6 \\
6y = -5x + 6 \\
y = -\frac{5x}{6} + 1
\]

The slope is \(-\frac{5}{6}\), and the \(y\)-intercept is 1.

**Example 4**

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**MANUFACTURING** Production figures for an assembly plant are represented by a line with a slope of \(\frac{1}{2}\) and a \(y\)-intercept of \(-1\). Find the equation of the line. Then draw the graph of the line.

**Solution**

\[
m = \frac{1}{2} \text{ and } b = -1
\]

\[
y = mx + b
\]

\[
y = \frac{1}{2}x - 1
\]

To draw the graph, start at point \((0, -1)\). Then using a slope of \(\frac{1}{2}\), locate a point 1 unit up and 2 units right at \((2, 0)\). Draw a line through the points.
TRY THESE EXERCISES

Find the slope of the line containing the given points.
1. \( A(3, 5) \) and \( B(0, 6) \)
2. \( P(-2, -6) \) and \( Q(2, 1) \)
3. \( G(-2, 1) \) and \( H(-2, 10) \)

Graph the line that passes through the given point \( P \) and has the given slope.
4. \( P(-1, 2) \), slope = -1
5. \( P(4, -1) \), slope = \( \frac{2}{3} \)

Find the slope and \( y \)-intercept of each line.
6. \(-5x + 13y = 15\)
7. \( y = -6x + 4 \)
8. \( y = 5x - 1 \)
9. \( y = -\frac{1}{10}x + \frac{1}{5} \)

Find an equation of the line with the given slope and \( y \)-intercept.
10. slope = -1, \( y \)-intercept = 3
11. slope = 4, \( y \)-intercept = -2

Graph each equation.
12. \(-2x + 3y = 3\)
13. \(2x - y = 4\)
14. \(y = 1\)

PRACTICE EXERCISES • For Extra Practice, see page 679.

Find the slope of each line.
15. [Graph of line]
16. [Graph of line]
17. [Graph of line]

18. Find the slope of the line containing the points \( A\left(\frac{1}{2}, \frac{-3}{4}\right) \) and \( B\left(\frac{2}{3}, \frac{5}{8}\right) \).

GRAPHING Plot the point \( P(1, 1) \). Then, on the same axes, graph the lines that pass through \( P \) and have the given slope.
19. \( m = 0 \)
20. \( m = 1 \)
21. \( m = -1 \)

DATA FILE For Exercises 22–24, use the data on the desired weight range based on corresponding height for men and women on page 650.
22. Use \( y \) as the lower male weight (in pounds) and \( x \) as the height (in inches). Plot the corresponding weights for \( x = 62, 63, 64 \). Connect the points. Label them \( P_1(62, w_1), P_2(63, w_2), \) and \( P_3(64, w_3) \).
23. Determine the slope for the line segment between \( P_1 \) and \( P_2 \) and between \( P_2 \) and \( P_3 \). Are the points collinear?
24. Repeat Exercise 23 using \( y \) as the lower female weight (in pounds) and \( x \) as the height (in inches). Plot \( 62, w_1), (63, w_2), (64, w_3) \). Connect the points.

Find the slope and \( y \)-intercept of each line.
25. \( y = 4x \)
26. \(-6x - y = 12 \)
27. \( x = -9 \)
28. \(-\frac{x}{2} + 2y = 0 \)
29. \( x = 2 \)
30. \(3y = -x + \frac{1}{2} \)
Write an equation of the line with the given slope and y-intercept.
31. \( m = -5, b = 4 \)  
32. \( m = 2, b = -\frac{3}{4} \)  
33. \( m = 6, b = 0 \)  
34. \( m = 0, b = -1 \)

**CALCULATOR** Graph each equation.
35. \( 2x + 3y = -6 \)  
36. \( -x + y = 3 \)  
37. \( y = -3x + 2 \)

**FINANCE** Dave receives a salary of $200 a week plus a commission of 10% of his weekly sales. An equation \( y = mx + b \) represents Dave’s weekly earnings. The y-intercept is Dave’s base salary. The slope of the line is his commission.

38. Write an equation representing Dave’s weekly earnings.
39. Graph the equation.
40. If Dave sells $1500 of goods for one week, what is his salary for the week?
41. What value of goods does Dave need to sell in one week to have weekly earnings of $500?

**EXTENDED PRACTICE EXERCISES**

42. **RECREATION** A ski resort is building a new ski slope. The desired slope of the new hill is 0.8. The horizontal distance from the crest of the hill to the bottom will be 400 ft. What should the height of the hill be?

Find the slope and y-intercept for each line.
43. \( y = ax + r - m \)  
44. \( \frac{b}{a}x + \frac{c}{b}y = 1 \)  
45. The standard form of an equation is \( Ax + By + C = 0 \). Write the slope-intercept form of the equation.

46. **WRITING MATH** How is each set of equations different?
   a. \( y = 2x \) and \( y = -2x \)  
   b. \( y = x - 3 \) and \( y = x + 3 \)

47. **YOU MAKE THE CALL** A line passes through \( P(-5, 2) \) and \( Q(-5, 5) \). Geoff says the slope of the line must be 0 because there is no change in \( x \). Does Geoff’s reasoning make sense?

**MIXED REVIEW EXERCISES**

Find each unit rate or unit price. Round answers to the nearest hundredth if necessary. (Lesson 5-1)
48. $3.87 for 18 oz of soup  
49. 496 mi in 8 h  
50. 487.5 ft in 325 steps  
51. $5.95 for 5 lb of nuts  
52. 426.4 mi on 13 gal of gas  
53. 9 mi in 2 h 15 min  
54. 128 cookies in 8 equal sacks  
55. $96.67 for 58 jars of jam  
56. 528 fish in 6 tanks

Find all the solutions for each equation. (Lesson 2-4)
57. \( |x| - 6 = 8 \)  
58. \( -24 = -3|x| \)  
59. \( |x| + 4 = 8 \)  
60. \( 3|x| = 21 \)  
61. \( \frac{1}{2}|x| = 13 \)  
62. \( 36 = 4|x| \)  
63. \( |x| - 7 = 8 \)  
64. \( -48 = -8|x| \)
Parallel and Perpendicular Lines

Goals
- Use slope to determine whether two lines are parallel or perpendicular.

Applications
- Electronics, Mapmaking, Manufacturing

**GRAPHING** Use the zoom feature of your graphing calculator to change the viewing window to a square. Then graph the following equations on the same screen.

\[
\begin{align*}
  y &= 2x - 1 \\
  y &= -\frac{1}{2}x + 1 \\
  y &= 2x \\
  y &= -\frac{1}{2}x + 2 \\
  y &= 2x + 1 \\
  y &= -\frac{1}{2}x - 2
\end{align*}
\]

Which lines seem to be parallel? Which seem to be perpendicular? What do you notice about the slopes and y-intercepts of parallel and perpendicular lines?

**BUILD UNDERSTANDING**

From the activity above, you can see that two lines with the same slope and different y-intercepts are parallel. Conversely, two lines are parallel if they have the same slope and different y-intercepts. Vertical lines are parallel and have slopes that are undefined.

Two nonvertical lines are perpendicular if the product of their slopes is \(-1\). Conversely, if the slopes of two lines are negative reciprocals of each other, then the lines are perpendicular.

**Example 1**

Find the slope \((m_1)\) of a line parallel to the given line and the slope \((m_2)\) of a line perpendicular to the given line.

a. The line containing points \(A(-2, 5)\) and \(B(0,-1)\).

b. The line containing points \(C(4,-1)\) and \(D(-5,-1)\).

c. \(x = 2\)

**Solution**

<table>
<thead>
<tr>
<th></th>
<th>(m)</th>
<th>(m_1)</th>
<th>(m_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>(-\frac{1}{2})</td>
<td>-3</td>
<td>(-\frac{1}{3})</td>
</tr>
<tr>
<td>b.</td>
<td>(-\frac{1}{5})</td>
<td>0</td>
<td>undefined</td>
</tr>
<tr>
<td>c.</td>
<td>undefined</td>
<td>undefined</td>
<td>0</td>
</tr>
</tbody>
</table>
Example 2

Determine whether each pair of lines is parallel, perpendicular, or neither.

a. \[7x + 2y = 14\]
\[7y = 2x - 5\]

b. \[-5x + 3y = 2\]
\[3x - 5y = 15\]

\[\frac{8}{3}x - 4y = 4\]

Solution

Rewrite each equation in slope-intercept form and find the slope of each line.

a. \[7x + 2y = 14\] \[\rightarrow y = -\frac{7}{2}x + 7\] \[m_1 = -\frac{7}{2}\]
\[7y = 2x - 5\] \[\rightarrow y = \frac{2}{7}x - \frac{5}{7}\] \[m_2 = \frac{2}{7}\]
\[m_1 \neq m_2\]
\[m_1 \cdot m_2 = -\frac{7}{2} \cdot \frac{2}{7} = -1\]
Because \(m_1 \cdot m_2 = -1\), the lines are perpendicular.

b. \[-5x + 3y = 2\] \[\rightarrow y = \frac{5}{3}x + \frac{2}{3}\] \[m_1 = \frac{5}{3}\]
\[3x - 5y = 15\] \[\rightarrow y = \frac{3}{5}x - 3\] \[m_2 = \frac{3}{5}\]
\[m_1 \neq m_2\]
\[m_1 \cdot m_2 = \frac{5}{3} \cdot \frac{3}{5} = 1 \neq -1\]
Because \(m_1 \neq m_2\) and \(m_1 \cdot m_2 \neq -1\), the lines are neither parallel nor perpendicular.

c. \[2x - 3y = 6\] \[\rightarrow y = \frac{2}{3}x - 2\] \[m_1 = \frac{2}{3}\]
\[\frac{8}{3}x - 4y = 4\] \[\rightarrow y = \frac{2}{3}x - 1\] \[m_2 = \frac{2}{3}\]
\[m_1 = m_2\]
Because \(m_1 = m_2\) and the \(y\)-intercepts are different, the lines are parallel.

Example 3

ELECTRONICS A manufacturer of circuit boards uses a grid system to insert connecting pins. The design for a board requires pins at points \(M(0, 1)\), \(N(3, 0)\), \(P(5, 6)\) and \(Q(2, 7)\). When the pins are connected, will \(MNPQ\) be a parallelogram?

Solution

Plot the points and draw the quadrilateral. Find the slope of \(\overline{NP}\) and \(\overline{MQ}\), and the slope of \(\overline{MN}\) and \(\overline{QP}\).

\[m_{NP} = \frac{6 - 0}{5 - 3} = \frac{6}{2} = 3\]; \[m_{MN} = \frac{0 - 1}{3 - 0} = -\frac{1}{3}\]

\[m_{MQ} = \frac{7 - 1}{2 - 0} = \frac{6}{2} = 3\]; \[m_{QP} = \frac{6 - 7}{5 - 2} = -\frac{1}{3}\]
Since the slopes of $\overline{NP}$ and $\overline{MQ}$ are the same, $\overrightarrow{NP} \parallel \overrightarrow{MQ}$. Because the slopes of $\overrightarrow{MN}$ and $\overrightarrow{QP}$ are the same, $\overrightarrow{MN} \parallel \overrightarrow{QP}$. Thus, opposite sides of the quadrilateral are parallel. Therefore, $\overline{MNPQ}$ is a parallelogram.

**TRY THESE EXERCISES**

Find the slope of a line parallel to the given line and a line perpendicular to the given line.

1. The line containing points $M(8, 3)$ and $N(-1, 5)$
2. The line containing points $A(0, 5)$ and $B(-6, 5)$
3. The line containing points $S(2, -1)$ and $T(4, 7)$
4. The line containing points $P(3, -5)$ and $Q(3, 2)$

**Determine whether each pair of lines is parallel, perpendicular, or neither.**

5. The line containing points $A(9, 5)$ and $B(-1, 6)$
   \[
   \text{The line containing points } C(-8, 2) \text{ and } D(12, 4)
   \]
6. $-6x - 14y = 5$
   \[
   7x = 3y + 21
   \]
7. $-5x + 2y = 10$
   \[
   3x - 5y = 15
   \]

8. **CARTOGRAPHY** A road atlas is laid out on a grid. A mapmaker notes intersections at points $A(1, 1)$, $B(2, 0)$, $C(3, 2)$ and $D(2, 3)$. If the points are connected, will $ABCD$ form a parallelogram?

**PRACTICE EXERCISES** • For Extra Practice, see page 680.

Find the slope of a line parallel to the given line and a line perpendicular to the given line.

9. The line containing $(6, -3)$ and $(4, 5)$
10. The line containing $(-3, 2)$ and $(-7, 8)$
11. $10x + 12y = -6$
12. $-3y + 2x = 7$
13. $y = -7x + 6$

**Determine whether each pair of lines is parallel, perpendicular, or neither.**

14. The line containing points $P(9, -4)$ and $Q(-2, 7)$
   \[
   \text{The line containing points } M(14, 8) \text{ and } N(19, 13)
   \]
15. $-4x + 6y = 6$
   \[
   5x - 2y = 10
   \]
16. $8y = 12x - 20$
   \[
   9x = 6y + 5
   \]

**MANUFACTURING** A machine is used to apply a heat-sensitive transfer to a product. The transfer is placed by locating three points on a grid. The points are $A(0, 0)$, $B(5, 3)$ and $C(7, -3)$. Do the connected points form a right triangle?
Determine whether each pair of lines is parallel, perpendicular, or neither.

18. \[1.5x - 4.5y = x + 15\] \[2 - 3x = 13 + \frac{1}{3}y\]

19. \[\frac{2}{3}y - \frac{1}{4}x = 1\] \[15 + 3x = 15 + 8y\]

Determine the value of \(x\) so that the line containing the given points is parallel to another line whose slope is also given.

20. \(A(x, 9)\) and \(B(-5, 6)\) slope = \(-2\)

21. \(M(-1, 7)\) and \(N(x, -1)\) slope = \(-\frac{1}{3}\)

Determine the value of \(y\) so that the line containing the given points is perpendicular to another line whose slope is also given.

22. \(A\left(-\frac{5}{6}, y\right)\) and \(B\left(\frac{2}{3}, 1\right)\) slope = \(-\frac{1}{3}\)

23. \(C\left(\frac{1}{3}, -\frac{2}{3}\right)\) and \(D(1, y)\) slope is undefined

24. **Writing Math** Given the coordinates of four points which are the vertices of a polygon, how can you determine if the polygon is a rectangle?

25. **Chapter Investigation** Brainstorm a list of products that the members of your group have used during the past week. Select a product that all agree is in need of improvement. Prepare a list of questions that could be used to find out whether consumers share your concerns.

**Extended Practice Exercises**

26. The vertices of a triangle are \(A(a, 0)\), \(B(a + b, c)\) and \(C(c + a + b, c - b)\). Determine if \(\triangle ABC\) is a right triangle.

Use your graph from Exercise 27 for Exercises 28–30.

27. Plot \(L(3, 4)\), \(I(9, 4)\), \(N(7, 1)\), and \(E(1, 1)\) on a coordinate plane.

28. Use what you know about the slopes of lines to show that quadrilateral \(\text{LINE}\) is a parallelogram.

29. Draw diagonals \(\overline{LN}\) and \(\overline{EI}\). Use the slopes of the diagonals to show whether they are perpendicular to each other.

30. On graph paper, draw two different shaped quadrilaterals in which the diagonals are perpendicular to each other. Name each type of quadrilateral. Give the slope of each side and each diagonal of each quadrilateral.

**Mixed Review Exercises**

Find the area of the shaded region of each figure. Use 3.14 for \(\pi\). Round to the nearest hundredth if necessary. (Lesson 5-2)

31. [Diagram of a triangle with sides 4 cm, 8 cm, and 8 cm.]

32. [Diagram of a circle with a radius of 10 cm.]

33. [Diagram of a rectangle with sides 3 cm and 0.5 cm, and a triangle with sides 2.5 cm and 2 cm.]
Review and Practice Your Skills

**Practice Lesson 6-1**

Find the slope of the line containing the given points.
1. A(2, 4) and B(1, 3)  
2. C(3, 2) and D(5, 6)  
3. E(0, 5) and F(4, 0)
4. G(–3, 5) and H(–5, 6)  
5. Z(6, –2) and Y(–3, 2)  
6. X(–2, –2) and W(–12, –8)
7. V(5, 7) and U(–4, 7)  
8. T(0, 0) and S(–6, 5)  
9. R(–2.4, 6) and Q(–1, –2.4)

Graph the line that passes through the given point P and has the given slope.
10. P(0, 1), m = 2  
11. P(–4, 1), m = –3  
12. P(–2, –1), m = \( \frac{4}{3} \)
13. P(–5, –3), m = 0  
14. P(–4, 2), m = \( \frac{2}{3} \)  
15. P(0, 0), m = –4

Find the slope and y-intercept for each line.
16. \( y = 3x + 2 \)  
17. \( y = –5x + 9 \)  
18. \( y = x \)  
19. \( y = \frac{2}{3}x – 1 \)  
20. \( x = –5 \)  
21. \( 2x + 3y = 12 \)  
22. \( 5x – 2y = 20 \)  
23. \( \frac{4}{3}x + 4y = 1 \)

Write an equation of the line with the given slope and y-intercept.
24. \( m = 4, b = –4 \)  
25. \( m = 0, b = 15 \)  
26. \( m = \frac{1}{3}, b = –2 \)  
27. \( m = \frac{7}{2}, b = –3 \)  
28. \( m = 2, b = \frac{1}{2} \)  
29. \( m = –20, b = 5 \)

Graph each equation.
30. \( y = x \)  
31. \( y = 2x \)  
32. \( y = x + 2 \)  
33. \( 5x – y = 10 \)  
34. \( x + 5y = 10 \)  
35. \( 5x – y = 0 \)
36. Write an equation of the line containing the points C(–5, 3) and D(5, 9).

**Practice Lesson 6-2**

Find the slope of a line parallel to the given line and a line perpendicular to the given line.
37. The line containing (4, –9) and (–3, 5)  
38. The line containing (8, 13) and (15, –6)
39. The line containing (–3, –6) and (5, 2)  
40. The line containing (–5, 10) and (–5, –17)
41. The line containing (2, –9) and (15, –14)  
42. The line containing (6, 6) and (–7, 7)
43. \( y = –3x + 10 \)  
44. \( y = –\frac{3}{5}x – 7 \)  
45. \( –12x – 5y = –1 \)

Determine whether each pair of lines is parallel, perpendicular, or neither.
46. The line containing (–3, –1) and (4, –3)  
47. The line containing (7, –2) and (–8, –2)
   The line containing (13, 9) and (27, 5)  
   The line containing (0, 14) and (0, –31)
48. \( y = \frac{1}{4}x – 6 \) and \( x + 4y = –8 \)  
49. \( y = \frac{7}{2}x + 17 \) and \( 7x – 2y = 28 \)
50. \( x + y = 10 \) and \( y = 4 – x \)  
51. \( \frac{1}{5}x + \frac{2}{5}y = –2 \) and \( 2y = x – 14 \)

Chapter 6 Linear Systems of Equations
Practice Lesson 6-1–Lesson 6-2

Find the slope of the line containing the given points. (Lesson 6-1)

52. $A(0, 6)$ and $B(-9, 0)$
53. $C(-3, 14)$ and $D(-3, 8)$
54. $E(-4, -8)$ and $F(-8, -4)$
55. $G(-725, 630)$ and $H(435, -240)$

Find the slope and $y$-intercept for each line. (Lesson 6-1)

56. $y = x - 4.5$
57. $y = -\frac{1}{4}x + 7$
58. $y = 8 - 2x$
59. $9x - 3y = 12$
60. $\frac{1}{2}x + 2y = 0$
61. $-\frac{5}{6}x + \frac{10}{3}y = 30$

Find the slope of a line parallel to the given line and a line perpendicular to the given line. (Lesson 6-2)

62. The line containing $(9, -4)$ and $(-5, 3)$
63. The line containing $(7, 12)$ and $(16, -7)$
64. The line containing $(-5, -6)$ and $(1, 0)$
65. The line containing $(-8, 13)$ and $(-8, -22)$
66. The line containing $(4, -18)$ and $(30, -28)$
67. The line containing $(6.8, 6.5)$ and $(2.8, 7)$

MathWorks Career – Precision Assemblers

Precision assemblers work in factories to produce manufactured goods. These workers must be able to follow complex instructions and complete many steps with very little error.

Precision assemblers work on the assembly of aircraft, automobiles, computers and many other electrical and electronic devices. They work closely with engineers to build prototypes.

Precision assemblers must know how to read engineering schematics and how to use specialized and precise measuring instruments. An incorrect measurement may cause a product to work improperly or to endanger its users.

1. You must be certain that the slope of an airplane wing is exactly 0.13. If the wing is 5 ft long, how many inches should it rise from the fuselage to the wing tip?

2. You are mounting a Global Positioning System (GPS) onto the dashboard of a car. The slope of the GPS must be identical to the slope of the dashboard in order to work properly. The dashboard has a depth of 18 in. wide and rises 6 in. Find the slope of the dashboard.

3. The GPS unit has a depth of $4\frac{3}{4}$ in. How much should it rise in order to match the slope of the dashboard?
Write Equations for Lines

Goals
Write equations for lines in slope-intercept and point-slope forms.

Applications
Product design, Fitness, Real Estate

Complete the table by matching the correct slope and x- and y-intercept with the linear equation.

<table>
<thead>
<tr>
<th></th>
<th>Equation</th>
<th>Slope</th>
<th>x-intercept</th>
<th>y-intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b.</td>
<td>( y = -\frac{1}{2} x + 2 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c.</td>
<td>-2</td>
<td></td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>d.</td>
<td>( y = 2x - 1 )</td>
<td></td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>e.</td>
<td></td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

**Example 1**

Write an equation of the line with a slope of \(-2\) and passes through the point \(P(-1, 3)\).

**Solution**

\[
\begin{align*}
    y - y_1 &= m(x - x_1) \\
    y - 3 &= -2(x - (-1)) \\
    y - 3 &= -2(x + 1) \\
    y - 3 &= -2x - 2 \\
    y &= -2x + 1
\end{align*}
\]

Point-slope form
Substitute for \(2\) for \(m\), \(-1\) for \(x_1\), and \(3\) for \(y_1\).
Solve for \(y\).
Slope-intercept form
You can also write an equation in point-slope form, if you know the coordinates of two points on the line.

**Example 2**

Write an equation of the line containing the points \( A(1, -3) \) and \( B(3, 2) \).

**Solution**

Given: \( x_1 = 1, y_1 = -3, x_2 = 3, y_2 = 2 \)

Find the slope: \( m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-3)}{3 - 1} = \frac{5}{2} \)

Find the equation using the point-slope form.

\[
\begin{align*}
y - y_1 &= m(x - x_1) \\
y - (-3) &= \frac{5}{2}(x - 1) & \text{Solve for } y. \\
y + 3 &= \frac{5}{2}x - \frac{5}{2} \\
y &= \frac{5}{2}x - \frac{11}{2}
\end{align*}
\]

An equation of the line is \( y = \frac{5}{2}x - \frac{11}{2} \).

You can write an equation for a line if you are given the graph of the line and are able to read information from the graph.

**Example 3**

**PRODUCT DESIGN** A technician is using a coordinate grid to design a schematic for a circuit board. A connection aligns with the line shown at the right. Write an equation of the line.

**Solution**

\( y \)-intercepts: The line intersects the \( y \)-axis at the point \( (0, 3) \). The \( y \)-intercept is 3.

\( y \)-intercepts: 

slope: Use two points on the line whose coordinates are easily determined.

Use \( (x_1, y_1) = (0, 3) \) and \( (x_2, y_2) = (2, -1) \).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 3}{2 - 0} = \frac{-4}{2} = -2
\]

An equation of the line: \( y = mx + b \)

\[y = -2x + 3 \quad \text{slope-intercept form} \]

**GRAPHING** When writing an equation from a graph, check your work using a graphing calculator. Using the graphing keys, enter the slope-intercept form of the equation and display the graph. You may need to adjust the size or shape of the viewing window.
Example 4

Write an equation of a line parallel to \( y = -\frac{1}{3}x + 1 \) and containing point \( R(1, 1) \).

Solution

\[ y = -\frac{1}{3}x + 1 \quad m = -\frac{1}{3} \]

Because parallel lines have equal slopes, \( m = -\frac{1}{3} \).

\[ y - y_1 = m(x - x_1) \quad \text{Point-slope form.} \]

\[ y - 1 = -\frac{1}{3}(x - 1) \quad x_1 = 1, y_1 = 1 \]

\[ y - 1 = -\frac{1}{3}x + \frac{1}{3} \]

\[ y = -\frac{1}{3}x + \frac{4}{3} \]

An equation of the line is \( y = -\frac{1}{3}x + \frac{4}{3} \).

Check Understanding

Write an equation of the line whose slope and point on the line are given.
1. \( m = \frac{3}{4}, P(-1, 6) \)
2. \( m = 0, R(2, -2) \)

Try These Exercises

Write an equation of the line with the given slope and \( y \)-intercept.

1. \( m = -3, b = -2 \)
2. \( m = -\frac{3}{5}, b = 0 \)

Write an equation of the line that passes through the given point with the given slope.

3. \( m = 7, Q(-1, -5) \)
4. \( m = \frac{3}{7}, S(5, 3) \)

Write an equation of the line containing the given points.

5. \( M(0, -1) \) and \( N(-1, 4) \)
6. \( T(-2, -3) \) and \( V(2, 2) \)
7. \( R(-6, 9) \) and \( S(9, 9) \)
8. \( A(-5, 6) \) and \( B(-5, 10) \)

Write an equation for the lines graphed below.

9.

10.

Practice Exercises • For Extra Practice, see page 681.

Write an equation of the line with the given slope and \( y \)-intercept.

11. \( m = \frac{2}{3}, b = -3 \)
12. \( m = 7, b = 2 \)

13. Write an equation of the line that is parallel to \( 3y + x = 6 \) containing the point \( P(1, -1) \).

14. **Temperature** The temperature of water at the freezing point is 0°C or 32°F. The temperature of water at the boiling point is 100°C or 212°F. Use these two data points to find an equation to convert the temperature from Celsius to Fahrenheit.
15. \( m = -\frac{1}{2}, B \left(0, -\frac{2}{5}\right) \)

16. \( m \) is undefined, \( D \left(\frac{1}{3}, \frac{1}{2}\right) \)

17. \( P(4, 0) \) and \( Q(-9, 3) \)

18. \( M(2, -5) \) and \( N(1, 3) \)

19. \( \begin{array}{c|c|c|c|c|c}
\hline
x & y & A & x & y & B \\
\hline
2 & 4 & & 8 & 2 & \ \\
\hline
\end{array} \)

21. **FITNESS** Kim’s average walking speed is 6 feet every 3 seconds. Complete the table. Then write an equation for the distance Kim walks in a given time.

<table>
<thead>
<tr>
<th>Time (t) sec</th>
<th>Distance Kim walks (d) feet</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>12</td>
</tr>
</tbody>
</table>

22. **TALK ABOUT IT** Marta is asked to write an equation of a line with a slope of 3 that contains point \( D(2, 5) \). Marta writes \( y - 2 = 3(x - 5) \), so \( y = 3x - 13 \). Where did Marta make her mistake?

23. The coordinates of the vertices of a parallelogram are \((2, 6), (7, 2), (3, 0), \) and \((-2, 4)\).

24. **WRITING MATH** Is the parallelogram a rhombus? Justify your answer.

25. **REAL ESTATE** A realtor is paid a fixed amount per week plus a commission on the total sales. If the fixed amount is \( F \) and the commission rate is \( p \% \), find an equation to represent the amount of pay he receives each week.

26. **MIXED REVIEW EXERCISES**

Find the surface area of each figure. Assume that the pyramid is a regular pyramid. Use 3.14 for \( \pi \). (Lesson 5-6)

27. \( 4 \text{ cm} \)

28. \( 7 \text{ cm}, 3.8 \text{ cm} \)

Refer to the diagram. Name each set using roster notation. (Lesson 1-3)

29. \( A \cup B \)

30. \( B \cup C \)

31. \( A' \cap C \)

32. \( B' \cap C' \)

33. \( A' \cap B' \)

34. \( A' \cup C' \)
Graph the following pairs of equations on the same coordinate plane. Then complete the table.

<table>
<thead>
<tr>
<th>Equations</th>
<th>Point of Intersection (x, y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (y = x ) and (y = 1)</td>
<td>( , )</td>
</tr>
<tr>
<td>2. (2y = x + 2) and (y = x)</td>
<td>( , )</td>
</tr>
<tr>
<td>3. (y = 2x + 1) and (y = x)</td>
<td>( , )</td>
</tr>
<tr>
<td>4. (y = x + 1) and (y = x)</td>
<td>( , )</td>
</tr>
</tbody>
</table>

Two linear equations with the same two variables form a **system of equations**. A **solution** of a system of equations is the ordered pair that makes both equations true. One way to solve a system of two equations is to graph both equations on the same coordinate plane. The **point of intersection** of the two lines is the **solution** of the system. If the graphs intersect in one point, the system is known as an **independent system**.

**Example 1**

Solve the system of equations by graphing. \(y = -2x + 1\)  
\(y = -3x + 4\)

**Solution**

Graph each equation.

\[
\begin{align*}
y &= -2x + 1 \quad b = 1, m = -2; \text{ use } (1, -1) \\
y &= -3x + 4 \quad b = 4, m = -3; \text{ use } (1, 1)
\end{align*}
\]

The lines intersect at the point \((3, -5)\).  
The solution of the system of equations is \((3, -5)\).

**Check**: Substitute \((3, -5)\) in each equation.

\[
\begin{align*}
y &= -2x + 1 \\
-5 &= -2(3) + 1 \\
-5 &= -6 + 1 \\
-5 &= -5 \, \checkmark
\end{align*}
\]

\[
\begin{align*}
y &= -3x + 4 \\
-5 &= -3(3) + 4 \\
-5 &= -9 + 4 \\
-5 &= -5 \, \checkmark
\end{align*}
\]

If the graphs of the equations do not intersect, the system is known as an **inconsistent system**. The lines are parallel and have no common points. There is **no solution** to this system of equations.
Example 2

Solve the system of equations by graphing. \( y = \frac{1}{2}x + 3 \)
\( 2y = x - 2 \)

Solution

Graph each equation, then read the solution from the graph.

\( y = \frac{1}{2}x + 3 \) \hspace{1cm} \( b = 3, m = \frac{1}{2}; \) use \((-2, 2)\)

\( y = \frac{1}{2}x - 1 \) \hspace{1cm} Rewritten in slope-intercept form.
\( b = -1, m = \frac{1}{2}; \) use \((2, 0)\)

The lines are parallel and do not intersect. Therefore, there is no solution.

If the graphs of the equations are the same line—i.e., the lines coincide—the system is a dependent system. Any point on the line is a solution. There are infinitely many solutions.

Example 3

Solve the system of equations by graphing. \( 4x + 2y = 8 \)
\( 3y = -6x + 12 \)

Solution

Graph each equation. Then read the solution.

\( 4x + 2y = 8 \) \hspace{1cm} Subtract 4x from both sides.
\( y = -2x + 4 \) \hspace{1cm} Divide both sides by 2.

\( 3y = -6x + 12 \)
\( y = -2x + 4 \) \hspace{1cm} Divide both sides by 3.

The equations are equivalent. The graphs are the same line. The system has an infinite number of solutions.

To summarize:

<table>
<thead>
<tr>
<th>If the graph of two lines:</th>
<th>then the system is:</th>
<th>and has:</th>
</tr>
</thead>
<tbody>
<tr>
<td>intersect in a point</td>
<td>independent</td>
<td>one solution</td>
</tr>
<tr>
<td>do not intersect</td>
<td>inconsistent</td>
<td>no solution</td>
</tr>
<tr>
<td>coincide</td>
<td>dependent</td>
<td>an infinite number of solutions</td>
</tr>
</tbody>
</table>

Equations can give you a better understanding of everyday life. For example, the rate of taxation is a function of the amount of taxable income earned. The table below shows the tax rate schedule for single individuals for 2003.

If you are single . . .

<table>
<thead>
<tr>
<th>Line</th>
<th>If taxable income</th>
<th>The tax is</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Is Over</td>
<td>But not over</td>
</tr>
<tr>
<td>1</td>
<td>$0</td>
<td>$7000</td>
</tr>
<tr>
<td>2</td>
<td>$7000</td>
<td>$28,400</td>
</tr>
<tr>
<td>3</td>
<td>$28,400</td>
<td>$68,800</td>
</tr>
</tbody>
</table>
You can graph the equations defined in the table to gain an understanding of how the tax law works. Let \( T \) represent the amount of income tax owed and \( x \) represent the amount of taxable income. Using these variables, construct three linear equations that reflect the information contained in the table. Notice that each equation has a different slope.

**Line 1:** If \( 0 < x \leq 7000 \), then \( T = 0.1x \)

**Line 2:** If \( 7000 < x \leq 28,400 \), then \( T = 700 + 0.15(x - 7000) \), or \( T = 0.15x - 350 \)

**Line 3:** If \( 28,400 < x \leq 68,800 \), then \( T = 3910 + 0.25(x - 28,400) \), or \( T = 0.25x - 3190 \)

**Example 4**

**INCOME TAX** Rick’s taxable income was $18,000. Find the amount of income tax he will owe.

**Solution**

His income is less than $28,400, so multiply by 0.15 and subtract 350.

\[
T = 0.15x - 350
\]

\[
= 0.15(18,000) - 350
\]

\[
= 2700 - 350
\]

\[
= 2350 \quad \text{Rick will owe $2350 in income tax.}
\]

**Try These Exercises**

Determine whether the given ordered pair is a solution.

1. \((2, 1)\) \(5y = 3x - 1\)
   \(2x - 3y = 1\)
2. \((-1, 3)\) \(7x + 2y = -1\)
   \(-4x + y = 15\)

Solve each system of equations by graphing.

3. \(x = -2y - 3\)
   \(x + 2y = -5\)
4. \(x = \frac{4}{3}y - 1\)
   \(4y = 3x + 3\)
5. \(-x + \frac{3}{2}y = 1\)
   \(4x = 2y + 4\)

**Practice Exercises** • For Extra Practice, see page 681.

Determine the solution of each system of equations.

6. 
7. 
GRAPHING  Solve each system of equations by graphing.

8. \[2x - y = 3\]
   \[x - \frac{1}{2}y = 1\]

9. \[4x - 2y = 4\]
   \[3x + 6y = 3\]

10. \[4y = 3x - 5\]
    \[x = 2y + 1\]

11. \[\frac{x}{3} + \frac{y}{5} = \frac{7}{15}\]
    \[\frac{x}{4} + \frac{3y}{8} = \frac{1}{8}\]

12. \[\frac{c}{3} = \frac{d}{4} + \frac{1}{8}\]
    \[\frac{3}{4}d = c + \frac{1}{2}\]

13. MANUFACTURING  The Food Division and Personal Care Division introduced a total of 10 new products this year. The Food Division introduced 4 more products than the Personal Care Division. How many products did each group introduce this year?

14. FINANCE  Candice’s monthly savings is twice the amount that she spends on transportation each month. The total of her monthly savings and transportation bill is $135. Find both amounts.

15. INCOME TAX  Phil’s taxable income was $48,000. Use the table on page 259 to find the amount of income tax he had to pay.

16. WRITING MATH  Is it possible for a system of equations to have exactly two solutions? Explain your thinking.

17. CHAPTER INVESTIGATION  Make a detailed drawing or build a prototype of your new product. Choose at least five selling points to emphasize in your marketing materials.

Determine the number of solutions for each system. Do not graph.

18. \[4x + 5y = 3\]
    \[3x - 2y = 8\]

19. \[\frac{a}{2} = \frac{b}{3} + \frac{1}{6}\]
    \[\frac{2}{3}b = a + \frac{1}{2}\]

20. \[y - 2 = \frac{1}{2}x\]
    \[\frac{x}{4} - \frac{1}{2}y = -1\]

EXTENDED PRACTICE EXERCISES

21. The graphs of the equations \(2x - y = 1\), \(x + 2y = 8\), and \(x = 4\) form a triangle. Graphically determine the location of each vertex.

22. Determine whether the ordered pair \((-a, \frac{b}{2})\) is a solution to the system of equations.
\[
4ay = -3bx - ab
\]
\[
5bx - 2ay = -6ab
\]

MIXED REVIEW EXERCISES

Find the probability that a point selected at random in each figure is in the shaded region.

23. \[\text{5 in.} \quad \text{1.2 in.} \quad \text{12 in.} \quad \text{8 in.}\]

24. \[\text{7 in.} \quad \text{2 in.} \quad \text{7 in.} \quad \text{3 in.}\]

25. \[\text{5 in.} \quad \text{1.6 in.} \quad \text{1.6 in.} \quad \text{5 in.}\]

Write each number in standard form.

26. \(3.84 \cdot 10^{-6}\)

27. \(1.9 \cdot 10^8\)

28. \(7 \cdot 10^{-9}\)

29. \(6.52 \cdot 10^{12}\)
Review and Practice Your Skills

**Practice Lesson 6-3**

**Write an equation of the line with the given slope and y-intercept.**

1. \( m = 4.5, b = -7.5 \)
2. \( m = -3, b = 16 \)
3. \( m = 0, b = -4 \)
4. \( m = \frac{1}{6}, b = 6 \)
5. \( m = \frac{5}{4}, b = 13 \)
6. \( m = \frac{11}{8}, b = -44 \)
7. \( m = 1, b = 0.05 \)
8. \( m = -9.25, b = 0 \)
9. \( m = 300, b = 530 \)

**Write an equation of the line that passes through the given point with the given slope.**

10. \( m = -\frac{2}{3}, G(0, 0) \)
11. \( m = 2, H(-4, -1) \)
12. \( m = \frac{3}{5}, J(5, 0) \)
13. \( m = \frac{7}{2}, K(2, 9) \)
14. \( m = -\frac{1}{2}, L(-8, 1) \)
15. \( m = -4, N(3, -6) \)
16. \( m = 0, P(-8, 9) \)
17. slope undefined, \( Q(13, -12) \)

**Write an equation of the line containing the given points.**

19. \( A(-8, 3) \) and \( B(4, 9) \)
20. \( C(2, -9) \) and \( D(-3, 11) \)
21. \( E(3, -5) \) and \( F(3, 7) \)
22. \( G(-5, 3) \) and \( H(10, -6) \)
23. \( J(2, 8) \) and \( K(-3, -7) \)
24. \( L(4, -2) \) and \( M(-12, -2) \)
25. \( P(-7, -8) \) and \( Q(6, 5) \)
26. \( R(12, -7) \) and \( S(-6, -4) \)
27. \( T(-4, -2) \) and \( U(8, 7) \)

**Write an equation for the line with the given information.**

28. parallel to \(-2x + y = -14\) and passes through \( Z(5, 3) \)
29. perpendicular to \(3y = -2x + 18\) and has \( x\)-intercept 8
30. parallel to the line through points \( G(-4, 8) \) and \( H(2, 5) \) and has same \( y\)-intercept as \( y = 8x - 11 \)

**Practice Lesson 6-4**

**Solve each system of equations by graphing.**

31. \( \begin{align*} y &= \frac{2}{3}x - 2 \\ x &= -3 \end{align*} \)
32. \( \begin{align*} y &= -\frac{1}{2}x + 4 \\ y &= 3x - 3 \end{align*} \)
33. \( \begin{align*} y &= 8 \\ x - y &= 0 \end{align*} \)
34. \( \begin{align*} y &= 2x - 4 \\ y &= \frac{1}{7}x + 9 \end{align*} \)
35. \( \begin{align*} 4y - 3x &= 12 \\ y &= \frac{3}{4}x - 5 \end{align*} \)
36. \( \begin{align*} y &= x + 2 \\ y &= -3x - 18 \end{align*} \)
37. \( \begin{align*} x - 3y &= -3 \\ 3y &= 2x - 6 \end{align*} \)
38. \( \begin{align*} y &= 5 - x \\ y &= 4x - 20 \end{align*} \)
39. \( \begin{align*} 3x + 12y &= -12 \\ 5x + 2y &= 16 \end{align*} \)

**Determine the number of solutions for each system. Do not graph.**

40. \( \begin{align*} y &= 3x - 8 \\ 3x - 4 &= -5 \end{align*} \)
41. \( \begin{align*} y &= -x + 4 \\ -3x + 2y &= 8 \end{align*} \)
42. \( \begin{align*} y &= \frac{1}{4}x - 1 \\ 2x - 8y &= 8 \end{align*} \)
43. The sum of two numbers is \(-3\). Their difference is \(13\). Find the numbers.
Practice  Lesson 6-1—Lesson 6-4

Find the slope and \( y \)-intercept for each line. (Lesson 6-1)

44. \( y = \frac{-1}{5}x + 4 \)  
45. \( y = x - \frac{3}{2} \)  
46. \( y = 15 - \frac{2}{3}x \)  
47. \( 6x - 4y = 28 \)  
48. \( 3x + 7y = -28 \)  
49. \( y = -12 \)

Determine whether each pair of lines is parallel, perpendicular, or neither. (Lesson 6-2)
50. \( y = 4x - 6 \) and \( 4x + y = 8 \)  
51. \( y = 7 - 2x \) and \( 4x + 2y = 16 \)  
52. \( y = -\frac{3}{8}x + 11 \) and \( 24x - 9y = 1 \)  
53. \( y = 16 \) and \( x = -3 \)

Write an equation for the line with the given information. (Lesson 6-3)
54. \( m = -\frac{1}{4}, B(0, -7) \)  
55. \( M(-1, -7), N(2, -10) \)  
56. \( m \) undefined, \( Z(-8, 6.5) \)  
57. \( m = 0, Q(-2.7, 36) \)  
58. \( m = 4, b = -11.4 \)  
59. \( m = 3, x \)-intercept \(-5 \)

Solve each system of equations by graphing. (Lesson 6-4)
60. \( y = -3x \) \( y = \frac{1}{2}x - 7 \)  
61. \( y = x - 9 \) \( y = \frac{4}{5}x \)  
62. \( x - 2y = 6 \) \( y = -\frac{3}{2}x + 5 \)

Mid-Chapter Quiz

Find the slope of the line containing the given points. (Lesson 5-1)
1. \( K(7, 1) \) and \( L(4, 7) \)  
2. \( I(2, 9) \) and \( J(-2, 6) \)

Write an equation of the line with the given slope and \( y \)-intercept. (Lesson 5-1)
3. \( m = -5, b = 3 \)  
4. \( m = \frac{3}{4}, b = -2 \)

Graph each equation. (Lesson 5-1)
5. \( x = -3 \)  
6. \( x + 2y = 4 \)

Determine whether each pair of lines is parallel, perpendicular, or neither. (Lesson 5-2)
7. \( 4x + y = 3 \) and \( x - 4y = -8 \)  
8. \( x + 2y = 4 \) and \( y = -2x + 2 \)
9. Plot and connect the points \( J(-3, 6), K(-5, 0), L(1, -2) \) and \( M(3, 4) \). Determine whether \( JKL \) is a rectangle.

Write an equation of each line. (Lesson 5-3)
10. containing the points \( Y(-3, -2) \) and \( Z(-1, 4) \)
11. that is perpendicular to \( x - 3y = 2 \) and contains the point \( (2, 4) \)  
12. that is parallel to \( 2x + 5y = 4 \) and contains the point \( (2, 1) \)

Solve each system of equations by graphing. (Lesson 5-4)
13. \( 3m + n = -8 \) \( m + 6n = 3 \)  
14. \( x + 2y = 5 \) \( -6y = 3x - 15 \)  
15. \( p - 2q = 4 \) \( 2q = p + 2 \)
Work with a partner to practice using the distributive property.
Write one of the equations below on a piece of paper. Pass the paper back and forth, adding the next line to solve the equation.

1. \(x + 2(3x - 6) = 2\)  
2. \(-4x - 2 = 2(x + 7)\)  
3. \(5(2x + 4) - 10 = 70\)  
4. \(3(2x + 9) = 81\)

Build Understanding
You can use algebraic methods to solve a system of equations. One of these methods is substitution. This method is useful when one equation has already been solved for one of the variables. In any algebraic method, you need to eliminate one variable so you will have an equation in one variable to solve.

Example 1
Find the solution to the system of equations. \(3x - y = 6\)  
\(x + 2y = 2\)

Solution
\[
\begin{align*}
3x - y &= 6 \\
y &= 3x - 6 & \text{Solve the first equation for } y \text{ in terms of } x. \\
x + 2y &= 2 & \text{Write the second equation.} \\
3x - 6 &= 2 & \text{Substitute } (3x - 6) \text{ for } y. \\
7x &= 14 & \text{Solve for } x. \\
x &= 2 & \\
\end{align*}
\]
Choose one of the original equations.
\[
\begin{align*}
3x - y &= 6 & \text{Substitute } 2 \text{ for } x. \\
3(2) - y &= 6 & \\
6 - y &= 6 & \text{Solve for } y. \\
y &= 0 & \\
\end{align*}
\]
\[
\begin{align*}
\text{Check } x &= 2, \ y = 0 \text{ in each original equation.} & \text{Check } x &= 2, \ y = 0 \\
3x - y &= 6 & x + 2y &= 2 \\
3(2) - 0 &= 6 & 2 + 2(0) &= 2 \\
6 - 0 &= 6 & 2 + 0 &= 2 \\
6 &= 6 & 2 &= 2 & \checkmark & \checkmark
\end{align*}
\]
The solution is \((2, 0)\).

The substitution method is most useful when one of the coefficients is 1 or \(-1\).
Example 2

Find the solution to the system of equations. \(2x + 3y = 6\)
\(4x + 6y = 6\)

Solution

\[
2x + 3y = 6 \quad 4x + 6y = 6
\]
\[
3y = -2x + 6 \quad \text{Solve for } y. \quad 4x + 6\left(-\frac{2}{3}x + 2\right) = 6 \quad \text{Substitute for } y.
\]
\[
y = -\frac{2}{3}x + 2 \quad 4x - 4x + 12 = 6
\]
\[
12 = 6
\]

There is no solution. The lines are parallel.

Example 3

Solve the system of equations. \(4x - 2y = 10\)
\(-2x + y = -5\)

Solution

\[
-2x + y = -5
\]
\[
y = 2x - 5 \quad \text{Solve for } y.
\]
\[
4x - 2y = 10
\]
\[
4x - 2(2x - 5) = 10 \quad \text{Substitute for } y.
\]
\[
4x - 4x + 10 = 10
\]
\[
10 = 10
\]

The lines are the same. There are infinitely many ordered pairs that satisfy both of the equations \(4x - 2y = 10\) and \(-2x + y = -5\).

Problems in everyday life can lead to a system of equations that can be solved using the substitution method.

Example 4

**SHIPPING** An appliance store delivers large appliances using vans and trucks. When loaded, each van holds 4 appliances and each truck holds 6. If 42 appliances are delivered and 8 vehicles are full, how many vans and trucks are used?

Solution

Define each of the variables.
Write and solve a system of equations relating to the variables.

Let \(t\) = number of trucks used
\(v\) = number of vans used

There are 8 vehicles.

\[
v + t = 8
\]
\[
t = 8 - v \quad \text{Solve for } t.
\]

42 appliances are delivered; 6 in each truck, and 4 in each van.
There are 3 vans and 5 trucks delivering appliances.

---

**Try These Exercises**

Solve and check each system of equations by the substitution method.

1. \(2x + y = 0\)
   \(x - 5y = -11\)

2. \(x + 3y = -9\)
   \(-5x - 2y = -7\)

3. \(x = \frac{1}{2}y\)
   \(-x + 6y = -11\)

4. \(x - 5y = 6\)
   \(y = -\frac{1}{2}x - \frac{1}{2}\)

5. **PACKAGING** The perimeter of a rectangular picture frame is 78 cm. If the width is \(\frac{3}{8}\) of the length, find the dimensions of the frame.

---

**Practice Exercises • For Extra Practice, see page 682.**

Solve and check each system of equations by the substitution method.

6. \(3x = y + 9\)
   \(2x - 4y = 16\)

7. \(-4x + 3y = -16\)
   \(-x + 2y = -4\)

8. \(\frac{x}{2} - y = \frac{5}{4}\)
   \(8x + 3y = 1\)

9. \(5x + 2y = 1\)
   \(3x + 4y = -5\)

10. \(6x - 3y = -9\)
    \(13x - 5y = -15\)

11. \(10x - 5y = 65\)
    \(10y - 5x = -55\)

12. **RECREATION** Jake is going on a 20-day vacation to the beach and to the mountains. He wants the time spent at the beach to be \(\frac{2}{3}\) the time spent in the mountains. How many days will he spend at the beach, and how many in the mountains?

13. **WRITING MATH** A friend asks you how to know which variable to solve for first when solving a system of equations by substitution. What advice would you give?

14. **FINANCE** Shari has 17 coins consisting of dimes and quarters worth $3.35. How many quarters and how many dimes does she have?

15. At the Hearty Hut, Chad bought 4 hamburgers and 5 fries and paid $8.71. Alisa bought 1 hamburger and 3 fries and paid $3.56. Find the cost of each hamburger and each order of fries.
Solve each system of equations by the substitution method. Check the solutions.

16. \[4y + 5x - 5 = 2x + 5y\]
   \[-9x + 2y = -4x - 8y\]

17. \[\frac{-3a + 5b}{2} = 1\]
   \[\frac{6a - 5b}{2} = 1\]

**DATA FILE** For Exercises 18–20, use the data on the shrinking value of the dollar on page 649. Use a system of equations to find the year in which these prices existed.

18. 3 qt of milk and 5 lb of round steak cost $5.33; 5 qt of milk and 1 lb of round steak cost $1.99

19. 20 lb of potatoes and 15 lb of flour cost $4.92; 10 lb of potatoes and 25 lb of flour cost $5.89

20. 5 lb of flour and 4 qt of milk cost $0.79; 15 lb of flour and 5 qt of milk cost $1.39

**EXTENDED PRACTICE EXERCISES**

Solve each system of equations by the substitution method. Check the solutions.

21. \[\frac{1}{x} + \frac{1}{y} = -1\]
   \[\frac{4}{x} - \frac{5}{y} = 23\]

22. \[\frac{2}{p} - \frac{1}{q} = -11\]
   \[\frac{-3}{p} - \frac{2}{q} = 13\]

23. \[-x + y - z = 4\]
   \[y = -x\]
   \[x - 4z = 7\]

24. \[-2a - 3b + c = 6\]
   \[c = a + 2b + 5\]
   \[a + 2b + 3c = -1\]

25. **FINANCE** Coins consisting of nickels, dimes, and quarters total $2.40. The number of dimes is equal to one less than \(\frac{2}{3}\) the number of nickels. Three times the number of quarters plus the number of dimes is 18. How many of each coin are there?

**MIXED REVIEW EXERCISES**

26. Find the volume of the figure shown. (Lesson 5-7)

Evaluate each expression when \(a = -4\) and \(b = 2\). (Lesson 1-8)

27. \(a^2 - b^2\)

28. \(a^2b^2\)

29. \((a - b)^2\)

30. \((a^2 + b^2)^2\)

31. \(a^3 - b^3\)

32. \(4(ab)^3\)

33. \(-3(a^3 - ab)^2\)

34. \(2(ab^2 + a^2b)\)

35. **CIVICS** In the United States, a president is elected every four years. Members of the House of Representatives are elected every two years and senators are elected every six years. If a voter had the opportunity to vote for a president, a representative, and a senator in 2004, what will be the next year the voter has a chance to make a choice for president, a representative, and the same seat in the Senate? (Prerequisite Skill)

35. Justin rented three times as many DVDs as Cole last month. Cole rented four fewer than Maria, but four more than Paloma. Maria rented 10 DVDs. How many DVDs did each person rent? (Prerequisite Skill)
Solve Systems by Adding and Multiplying

Goals

- Solve systems of equations by adding, subtracting, and multiplying.

Applications

- Entertainment, Community Service, Manufacturing

Work with a partner and use Algeblocks to simplify these expressions.

a. \((3x - y - 6) - (3x - 2y - 6)\)

b. \((4x + y + 4) + (2x - y - 10)\)

Build Understanding

Another algebraic method for solving a system of equations is the addition/subtraction method. To eliminate one of the variables to get one equation in one variable, you can add or subtract the two equations. If the coefficients of one of the variables are opposites or the same, simply adding or subtracting the equations eliminates one of the variables.

Steps to follow when using the addition/subtraction method:

1. If the coefficients of one of the variables are opposites, add the equations to eliminate one of the variables.
   If the coefficients of one of the variables are the same, subtract the equations to eliminate one of the variables.

2. Solve the resulting equation for the remaining variable.

3. Substitute the value for the variable in one of the original equations and solve for the other variable.

4. Check the solution in both of the original equations.

Example 1

Solve: 
\[
\begin{align*}
2x + 7y &= -5 \\
-5x + 7y &= -12
\end{align*}
\]

Solution

The \(y\)-coefficients are the same, so subtract the equations.

\[
\begin{align*}
2x + 7y &= -5 & \rightarrow & & 2x + 7y &= -5 \\
-5x + 7y &= -12 & \text{Add.} & & 5x - 7y &= 12
\end{align*}
\]

\[
\begin{align*}
7x &= 7 \\
x &= 1
\end{align*}
\]

Choose one of the original equations.

\[
\begin{align*}
2x + 7y &= -5 \\
2(1) + 7y &= -5 & \text{Substitute for } x.
\end{align*}
\]

\[
\begin{align*}
y &= -1 \\
7y &= -7 & \text{The check is left to you. The solution is } (1, -1).
\end{align*}
\]
Unless the coefficients of one variable are the same or are opposites, you will still have an equation with two variables when you add or subtract the equations. In that case, you first need to multiply one or both of the equations by a number to obtain an equivalent system of equations in which the coefficients of one of the variables are the same or opposite. Then add or subtract.

This method combines the multiplication property of equations with the addition/subtraction method, and is known as the **multiplication and addition method**.

Both the addition/subtraction method and the multiplication and addition method are best used when the equations are written in standard form.

**Example 2**

Solve: \(3x - 4y = 10\)
\(3y = 2x - 7\)

**Solution**

\[
\begin{align*}
3x - 4y &= 10 \\
-2x + 3y &= -7
\end{align*}
\]

Rewrite the second equation in standard form.

To eliminate \(x\), multiply the first equation by 2 and the second by 3. Then add.

\[
\begin{align*}
3x - 4y &= 10 \\
-2x + 3y &= -7
\end{align*} \rightarrow \begin{align*}
6x - 8y &= 20 \\
9x - 9y &= -21
\end{align*}
\]

Add. \(y = -1\)

\[
\begin{align*}
3x - 4y &= 10 \\
3x - 4(-1) &= 10
\end{align*}
\]

Choose one of the original equations.

\[
\begin{align*}
3x + 4 &= 10 \\
3x &= 6 \\
x &= 2
\end{align*}
\]

The check is left to you. The solution is \((2, -1)\).

**Example 3**

**ENTERTAINMENT** Tim sold 25 movie tickets for a total of $132. If each adult ticket sold for $6 and each children’s ticket sold for $4, how many of each kind did he sell?

**Solution**

Make a chart for the number and values of the tickets.

<table>
<thead>
<tr>
<th></th>
<th>Adult</th>
<th>Child</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>(A)</td>
<td>(C)</td>
<td>(A + C)</td>
</tr>
<tr>
<td>Value</td>
<td>6(A)</td>
<td>4(C)</td>
<td>6(A) + 4(C)</td>
</tr>
</tbody>
</table>

Write and solve a system of equations to represent the problem.
The number of tickets sold is 25.  \( A + C = 25 \)
The value of the tickets is $132.  \( 6A + 4C = 132 \)

\[
\begin{align*}
A + C = 25 & \quad \Rightarrow -4A - 4C = -100 \\
6A + 4C = 132 & \quad \Rightarrow 6A + 4C = 132
\end{align*}
\]

Add.  \( \Rightarrow 2A = 32 \)
\( A = 16 \)

\( A + C = 25 \)  Choose one of the original equations.
\( 16 + C = 25 \)  Substitute for \( A \).
\( C = 9 \)

Tim sold 16 adult tickets and 9 children's tickets.

**Try These Exercises**

Solve each system of equations. Check the solutions.

1. \( x - y = -1 \)
\( x + y = 9 \)

2. \( 3x + y = -7 \)
\( 5x - y = -9 \)

3. \( -3x - 5y = 4 \)
\( 2x + y = -5 \)

4. \( 4x = 2y - 12 \)
\( 3y = x + 13 \)

5. **COMMUNITY SERVICE**  Adult tickets for a benefit breakfast cost $2.50. Children's tickets cost $1.50. If 56 tickets were sold for total sales of $97, how many of each kind were sold?

6. **FINANCE**  Afton invested $5400 in two products, a new mouthwash and a new line of frozen dinners. She invested \( \frac{1}{2} \) as much money in mouthwash as she did in frozen dinners. How much did she invest in each product?

7. **MANUFACTURING**  It takes 18 months from the time a shoe design is approved for the shoes to arrive in stores for sale. The actual assembly of the shoe takes 3 months less than the time spent on development. How many months does the assembly take?

8. **WRITING MATH**  Using the multiplication and addition method, how can you know when a system has an infinite number of solutions?

**Practice Exercises**  •  For Extra Practice, see page 682.

Solve each system of equations. Check the solutions.

9. \( -x + 2y = 3 \)
\( 3x + 2y = -1 \)

10. \( x = 5y + 7 \)
\( 5y = 9x + 15y - 8 \)

11. \( -2x + 6y = 10 \)
\( 2x - 9y = -19 \)

12. \( -10x + 6y = 25 \)
\( 9y = -2x + 12 \)

13. \( 4x - 9y = -1 \)
\( 9y = 8x + 1 \)

14. \( 7x + 4y = 6 \)
\( 5y + 7x = 15 \)

15. \( 8x + 3y = -4 \)
\( 6x + 5y = 8 \)

16. \( 12x - 3y = 18 \)
\( 8x = 2y + 12 \)

17. **FARMING**  The Deckerts grow wheat and barley on their 1200-acre farm. The amount of wheat they plant is 200 acres more than 3 times the number of acres of barley. How many acres of wheat and how many acres of barley do they plant?

18. Jason has 15 coins in his pocket, consisting of nickels and dimes. The total value of the coins is $1.15. How many of each coin does Jason have?
Solve each system of equations. Check the solutions.

19. \( \frac{x}{2} - \frac{4y}{3} = -3 \) \hspace{1cm} 20. \( 2x - 5 = 3y - 7 \) 
   \(-3x + 4y = 6 \) \hspace{1cm} 6y + 4 = 5x + 6

22. \( \frac{1}{3}a - \frac{1}{2}b = -9 \) \hspace{1cm} 23. \( \frac{5x + 3y}{10} = \frac{-1}{2} \)
   \( \frac{1}{5}b = \frac{1}{4}a + 5 \) \hspace{1cm} \( \frac{9x}{16} + \frac{7y}{25} = \frac{-17}{20} \)

24. \( \frac{s}{2} - \frac{r}{4} + \frac{3}{4} = \frac{-1}{2} \) \hspace{1cm} 25. A number divided by 3 plus another number divided by 9 have a sum of 7.
   If the first number is multiplied by 4 and divided by 5 and then subtracted \textit{from} the second number divided by 2, the result is \(-3\). What are the numbers?

\section*{Extended Practice Exercises}

Solve each system of equations. Check the solutions.

26. \( \frac{9}{x} + \frac{4}{y} = -19 \) \hspace{1cm} 27. \( \frac{7}{s} = \frac{2}{r} + \frac{2}{15} \)
   \( -\frac{6}{x} - \frac{2}{y} = 14 \) \hspace{1cm} \( \frac{5}{r} = \frac{3}{s} + \frac{5}{14} \)

28. \( 3x - 2y + z = 9 \) \hspace{1cm} 29. \(-a + 3b - 2c = 9 \)
   \( 2x + y - z = 2 \) \hspace{1cm} \( 9a + 4b = 2c + 1 \)
   \( 4x + 5y + 2z = 5 \) \hspace{1cm} \( 12a + 4b + 8c = -12 \)

30. \( y = ax^2 + bx + c \) is an equation for a quadratic function. \((0, 0), (1, -2)\) and \((2, 3)\) are solutions of the equation. Find \(a, b,\) and \(c\).

\section*{Mixed Review Exercises}

Complete the table. Use 3.14 for \(\pi\). Assume each planet is a sphere. Round answers to the nearest million. (Lesson 5-6)

<table>
<thead>
<tr>
<th>Planet</th>
<th>Diameter at equator</th>
<th>Radius</th>
<th>Surface area</th>
</tr>
</thead>
<tbody>
<tr>
<td>31. Mercury</td>
<td>3031 mi</td>
<td>1515.5 mi</td>
<td></td>
</tr>
<tr>
<td>32. Mars</td>
<td>4200 mi</td>
<td></td>
<td></td>
</tr>
<tr>
<td>33. Saturn</td>
<td>71,000 mi</td>
<td></td>
<td></td>
</tr>
<tr>
<td>34. Uranus</td>
<td>32,000 mi</td>
<td></td>
<td></td>
</tr>
<tr>
<td>35. Neptune</td>
<td>30,600 mi</td>
<td></td>
<td></td>
</tr>
<tr>
<td>36. Pluto</td>
<td>715 mi</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Solve each inequality. Graph the solution set on the number line. (Lesson 2-6)

37. \( 7 - x > -3 \) \hspace{1cm} 38. \( 6 + 5x \leq 11 \) \hspace{1cm} 39. \( x - 4 < -8 \)
   40. \( 8 - x \geq 4 \) \hspace{1cm} 41. \( 3x - 5 < 13 \) \hspace{1cm} 42. \( 4(x + 3) \geq -4 \)
   43. \( \frac{1}{2}(x - 4) > 3 \) \hspace{1cm} 44. \( 2(3 - x) \leq -1 \)

\section*{Space}

For Exercises 45 and 46, use the following information.

Objects weigh six times more on Earth than they do on the moon because the force of gravity is greater. (Prerequisite Skill)

45. Write an expression for the weight of an object on Earth if its weight on the moon is \(x\).

46. A scientific instrument weighs 34 lb on the moon. How much does the instrument weigh on Earth?
Review and Practice Your Skills

**PRACTICE ▶ Lesson 6-5**

Solve and check each system of equations by the substitution method.

1. \( x = 5y \)
   \( x - 3y = 6 \)

2. \( m = -2n - 2 \)
   \( 3m - 2n = 10 \)

3. \( x - y = 1 \)
   \( y = -x + 5 \)

4. \( y + 6 = 3x \)
   \( 9x - 2y = 3 \)

5. \( 3x - 2y = -3 \)
   \( 3x + y = 3 \)

6. \( 3p - 4q = 8 \)
   \( 4p + q = 17 \)

7. \( x - 2y = 16 \)
   \( 4x + y = 1 \)

8. \( s + 2t = 6 \)
   \( 4s + 3t = 4 \)

9. \( y = -4x + 5 \)
   \( 2x - 3y = 13 \)

10. \( 5n - v = -23 \)
    \( 3n - v = -15 \)

11. \( -2x + y = 2 \)
    \( 2x + 3y = 6 \)

12. \( x + 5y = 11 \)
    \( 4x - y = 2 \)

13. \( 2x + 3y = 11 \)
    \( 3x + 3y = 18 \)

14. \( 5a + 3b = 4 \)
    \( 4a - 2b = 1 \)

15. \( 5x + 7y = -3 \)
    \( 2x + 14y = 2 \)

16. Leonard has 23 coins consisting of quarters and nickels worth $4.15. How many quarters and how many nickels does he have?

17. Lisa bought 7 bagels and 4 peaches and paid $5.25. Emily bought 1 bagel and 6 peaches and paid $2.65. Find the cost of each bagel and each peach.

**PRACTICE ▶ Lesson 6-6**

Solve each system of equations. Check the solutions.

18. \( x - y = -5 \)
    \( x + y = 1 \)

19. \( 2r + s = -1 \)
    \( -2r + s = 3 \)

20. \( x + y = 5 \)
    \( x + 2y = 8 \)

21. \( 3m + n = -6 \)
    \( m - n = -2 \)

22. \( 3x + 4y = 8 \)
    \( 5x - 4y = 24 \)

23. \( x + 2y = 0 \)
    \( x - y = -3 \)

24. \( 8x - 3y = 17 \)
    \( -7x + 6y = 2 \)

25. \( 7p - 10q = -1 \)
    \( 3p + 2q = -13 \)

26. \( 4x - 3y = 15 \)
    \( 8x + 2y = -10 \)

27. \( 2a + 8b = -1 \)
    \( -10a + 4b = 16 \)

28. \( 5x + 3y + 9 = 0 \)
    \( 3x - 4y + 17 = 0 \)

29. \( 6g - 5 = 2g - 7h \)
    \( 2g = 5h - 6 \)

30. \( 6x + 7y = -11 \)
    \( 4x - 7y = 21 \)

31. \( 3c + 4d = -15 \)
    \( 3c + 9d = 10 \)

32. \( 3x - 5y = -1 \)
    \( 6x + 2y = 10 \)

33. \( -6x + 2y = -10 \)
    \( -5x + 7y = -35 \)

34. \( 3x + 3y + 4 = 0 \)
    \( 9x - 5y = -20 - 6x \)

35. \( 3x - y = 15 \)
    \( y = 3(x - 7) \)

**PRACTICE ▶ Lesson 6-1–Lesson 6-6**

Graph the line that passes through the given point \( P \) and has the given slope.

(Lesson 6-1)

36. \( P(-4, 5), m = -\frac{1}{2} \)  

37. \( P(0, 6), m = \frac{2}{3} \)  

38. \( P(-7, -1), m = 0 \)  

39. \( P(4, -8), m = 3 \)  

40. \( P(9, 7), m = -5 \)  

41. \( P(1.5, 1.5), m = 1.5 \)
Determine whether each pair of lines is parallel, perpendicular, or neither. (Lesson 6-2)

42. The line containing \( A(0, 5) \) and \( B(-3, 7) \)
43. The line containing \( C(-4, -9) \) and \( D(5, 9) \)

The line containing \( Y(10, 2) \) and \( Z(4, 6) \)
The line containing \( M(-7, -1) \) and \( N(1, -5) \)

Write an equation for the line with the given information. (Lesson 6-3)

44. \( y = 4.5x - 6.3 \)
45. \( y = -13 \)
46. \( 3x + y = 0 \)

18x - 4y = 36
\( y = -13x \)
3y = x

Solve each system of equations by graphing. (Lesson 6-4)

53. \( x + y = 2 \)
54. \( x + y = 1 \)
55. \( 3x + 2y = -8 \)

\( y = x - 4 \)
\( y = x + 5 \)
\( 2x - 3y = -1 \)

---

**Career – Engineering Technician**

Engineering technicians help design and build new products. They use mathematics, engineering, and science to solve technical problems. They may be asked to create specifications for materials, establish quality testing procedures, and improve manufacturing efficiency. Engineering technicians work in laboratories, offices, industrial plants, and construction sites. They use math to make precise measurements and create schematics. They must be able to apply mathematical reasoning to find ways to cut costs and save time.

1. You are testing two products for your company. Your boss gives you a budget of $800 to spend on the testing. You ended up spending \( \frac{1}{4} \) as much on product A as on product B. If you spent the whole amount, how much did you spend on each product?

2. You study efficiency in your plant by using the formula:

\[
\text{efficiency rate} = \frac{\# \text{ of products}}{\# \text{ of hours}}
\]

You discover that the efficiency rating for the second shift at the plant is 61.125 for 7 \( \frac{1}{2} \) h of work. How many products are they producing each day?

3. After research, you discover that the efficiency rating of the second shift increases by 1.75 points when the workers are given three 15-min breaks instead of only two. Recalculate the number of products the shift produces based on the loss of 15 min from the shift but a gain of 1.75 in efficiency rating. Should the plant increase the number of breaks?
Another method of solving a system of equations is the method of determinants. A 2 × 2 determinant is a square array in the form
\[
\begin{vmatrix}
 a & b \\
 c & d
\end{vmatrix}
\]
consisting of two rows and 2 columns.

The **determinant of a system of equations**, det \( A \), is formed using the coefficient of the variables when the equations are written in standard form.

\[
\begin{align*}
\text{System of equations} & \quad \begin{cases} 
ax + by = e \\
rx + dy = f
\end{cases} \quad \text{← det} \ A = \begin{vmatrix}
 a & b \\
 c & d
\end{vmatrix} \quad \text{Determinant of coefficients}
\end{align*}
\]

The value of the determinant is given by det \( A = ad - bc \), which is the difference of the product of the diagonals.

You can find the solution to a system of equations using this determinant and another determinant formed by replacing the \( x \) or \( y \) column of the determinant with the constant column.

\[
\begin{align*}
Ax &= \begin{vmatrix}
 e & b \\
 f & d
\end{vmatrix} \quad \text{Replace the} \ x\text{-column with the constant column.} \\
Ay &= \begin{vmatrix}
 a & e \\
 c & f
\end{vmatrix} \quad \text{Replace the} \ y\text{-column with the constant column.}
\end{align*}
\]

To find \( x \), divide \( Ax \) by determinant \( A \).

\[
x = \frac{Ax}{A} = \frac{\begin{vmatrix}
 e & b \\
 f & d
\end{vmatrix}}{\begin{vmatrix}
 a & b \\
 c & d
\end{vmatrix}} = \frac{ed - bf}{ad - bc}, \quad A \neq 0
\]

To find \( y \), divide \( Ay \) by determinant \( A \).

\[
y = \frac{Ay}{A} = \frac{\begin{vmatrix}
 a & e \\
 c & f
\end{vmatrix}}{\begin{vmatrix}
 a & b \\
 c & d
\end{vmatrix}} = \frac{af - ec}{ad - bc}, \quad A \neq 0
\]

A **matrix** is an array of numbers. Each number in the array is called an **element**. A **column matrix** is an array of only one column. A **row matrix** is an array of only one row. A **square matrix** is an array with the same number of rows and columns.

Examples are:

- Column matrix: \( \begin{bmatrix} 2 \\ 1 \end{bmatrix} \), Row matrix: \([ 1 \ 2] \), Square matrix: \( \begin{bmatrix} 2 & 1 \\ -1 & 5 \end{bmatrix} \)

A system of equations \( 2x - y = 4 \) \quad 
\quad \text{can be written in matrix form by using}
\[
-3x + 2y = 5
\]
a square matrix and 2 column matrices.

The matrix equation is \( AX = B \) \quad \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}

**Problem**

Solve by the method of determinants.

\[
\begin{align*}
x + 3y &= 4 \\
-2x + y &= -1
\end{align*}
\]
Solution

\[ x = \frac{A_x}{A} = \frac{4 \ 3}{-1 \ 1} = \frac{4(1) - (3)(-1)}{1(1) - 3(-2)} = \frac{4 + 3}{1 + 6} = \frac{7}{7} = 1 \]

\[ y = \frac{A_y}{A} = \frac{1 \ 4}{-2 \ -1} = \frac{1(-1) - (4)(-2)}{1(1) - 3(-2)} = \frac{-1 + 8}{1 + 6} = \frac{7}{7} = 1 \]

The solution is (1, 1). The check is left to you.

**TRY THESE EXERCISES**

Write the matrix equations for Exercises 1–5. Then for each of the systems of equations, calculate \( \det A \), the determinant of the coefficients, and find the solution of the system using determinants. For Exercises 4 and 5, also define the system of equations.

1. \( 5x + y = 6 \)
   \[ -3x + 4y = 2 \]
2. \( 3x - y = 2 \)
3. \( 4x - 7y = 2 \)
4. **FINANCE** Deanna has $2.15 in dimes and quarters. If the dimes were nickels and the quarters were dimes, she would have $1.25 less. How many of each coin does Deanna have?
5. **BUSINESS** Car Rental Company A charges $25 per day plus $0.35 per mile. Company B charges $35 per day, plus $0.25 per mile. Wayne Know-it-all determines the cost of a trip he will take will be $230 for Company A and $250 for Company B. How many miles and for how many days will Mr. Know-it-all’s trip be?
6. **WRITING MATH** If \( \det A = 0 \), what is the solution of a system of equations?

**MIXED REVIEW EXERCISES**

Find the perimeter of each figure. (Lesson 5-2)

7. 

8. 

9. 

10. 

11. 

12. 

---

Lesson 6-7 Problem Solving Skills: Determinants and Matrices
Systems of Inequalities

**Goals**
- Use graphing to solve systems of linear inequalities.

**Applications**
- Manufacturing, Health, Budgeting

**Work with a partner.**

1. On a coordinate plane, graph \( y = x + 1 \) and \( y = x - 1 \).

2. Plot each point listed in the table. For each point, replace \( \leq \) with \(<, =, \text{ or } >\).

3. On the graph, shade the region where \( y < x + 1 \).

4. Shade the region of the graph where \( y > x - 1 \).

5. Find the section on the graph where the shading overlaps. What conclusion can you draw about this area?

<table>
<thead>
<tr>
<th>Point</th>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
<tbody>
<tr>
<td>((0, 0))</td>
<td>(y \leq x + 1)</td>
<td>(y \geq x - 1)</td>
</tr>
<tr>
<td>((1, 1))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>((1, -1))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>((-1, 1))</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Build Understanding**

The activity above shows a system of linear inequalities. A **system of linear inequalities** can be solved by graphing each related equation and determining the region where each inequality is true. The intersection of the graphs of the inequalities is the **solution set** of the system.

**Example 1**

Determine whether the given ordered pair is a solution to the given system of inequalities.

a. \((3, 1); x + 2y < 5\)
   \(2x - 3y \leq 1\)

b. \((2, -5); 4x - y \geq 5\)
   \(8x + 5y \leq 3\)

c. \((1, 2); x + y \geq 3\)
   \(3x - y < 1\)

**Solution**

Substitute for \(x\) and \(y\) in each system of inequalities.

a. \(x = 3, y = 1\)
   \(x + 2y < 5\)
   \(2x - 3y \leq 1\)
   \(3 + 2 < 5\) False
   \(6 - 3 \leq 1\) False

The ordered pair is not a solution for either inequality. Therefore, \((3, 1)\) is not a solution of this system.

b. \(x = 2, y = -5\)
   \(4x - y \geq 5\)
   \(8 + 5 \geq 5\)
   \(13 \geq 5\) True
   \(8 + 5 \leq 3\)
   \(16 - 25 \leq 3\)
   \(-9 \leq 3\) True

The ordered pair is a solution for both inequalities. Therefore, \((2, -5)\) is a solution of this system.
c. \(x = 1, \ y = 2\) 
\[\begin{align*}
  x + y &\geq 3 \\
  1 + 2 &\geq 3 \\
  3 &\geq 3 \quad \text{True} \\
  3x - y &< 1 \\
  3 - 2 &< 1 \\
  1 &< 1 \quad \text{False}
\end{align*}\]

The ordered pair is a solution for only one of the inequalities. Therefore, \((1, 2)\) is not a solution of this system.

**Example 2**

Write a system of linear inequalities for the graph at the right.

**Solution**

<table>
<thead>
<tr>
<th>1. Determine the equation of each line</th>
<th>2. Determine shading</th>
<th>3. Determine inequality symbol</th>
<th>4. Inequality</th>
</tr>
</thead>
<tbody>
<tr>
<td>line (l:) (b = -2, \ m = 1) (y = x - 2)</td>
<td>below and including line</td>
<td>(\leq)</td>
<td>(y \leq x - 2)</td>
</tr>
<tr>
<td>line (m:) (b = 0, \ m = -1) (y = -x)</td>
<td>below line</td>
<td>(&lt;)</td>
<td>(y &lt; -x)</td>
</tr>
</tbody>
</table>

The system of linear inequalities for the graph is
\[y \leq x - 2\]
\[y < -x\]

**Example 3**

**MANUFACTURING** A company writes a system of inequalities, shown below, to analyze how changes in plastic and paper affect a product's cost. Graph the solution set of the system.

\[\begin{align*}
  2x - 3y &\leq 6 \\
  x + 2y &< 2
\end{align*}\]

**Solution**

First graph the related equation for each inequality. Write each inequality in slope-intercept form. Then make a chart to use for graphing.

\[\begin{align*}
  2x - 3y &\leq 6 \\
  -3y &\leq -2x + 6 \\
  y &\geq \frac{2}{3}x - 2
\end{align*}\]

\[\begin{align*}
  x + 2y &< 2 \\
  2y &< -x + 2 \\
  y &< -\frac{1}{2}x + 1
\end{align*}\]

The solution set consists of all the points in the region that has been doubly shaded. The solution set includes points on the solid boundary line \(y = \frac{2}{3}x - 2\), but not on the dashed boundary line \(y = -\frac{1}{2}x + 1\).
TRY THESE EXERCISES

Determine whether the given ordered pair is a solution of the given system of inequalities.

1. \((1, -3); 3x + 4y \leq 12\)
   
   \(-5x + y \leq 5\)

2. \((-2, 1); 2x + y < 4\)
   
   \(2x - 2y \geq 3\)

Write a system of linear inequalities for the given graph.

3. \[\begin{align*}
   y &\leq 5 \\
   y &\geq 1
\end{align*}\]

4. \[\begin{align*}
   y &< x \\
   x + y &\geq 2
\end{align*}\]

Graph the solution set of the system of linear inequalities.

5. \[\begin{align*}
   x &\leq 5 \\
   y &\geq 1
\end{align*}\]

6. \[\begin{align*}
   x &< y \\
   x + y &\geq 2
\end{align*}\]

7. \[\begin{align*}
   2x - y &\leq 2 \\
   x + y &> -1
\end{align*}\]

8. \[\begin{align*}
   4 &< 3x - y \\
   y &> 2x - 1
\end{align*}\]

Technology Note

Most graphing calculators allow you to shade portions of a graph. Perform these steps on the \(Y=\) screen.

1. Write the inequality in slope-intercept form and enter as an equation.

2. If the inequality contains \(<\) or \(>\) symbols, change the display to show a dashed line.

3. Determine which side of the line must be shaded and choose a shading option.

PRACTICE EXERCISES • For Extra Practice, see page 682.

Determine whether the given ordered pair is a solution of the given system of inequalities.

9. \((3, 5); x - y \leq -4\)
   
   \(x - 2y \leq 1\)

10. \((-2, -1); x + 3y \leq 6\)
   
   \(4x - 2y \geq 4\)

Write a system of linear inequalities for the given graph.

11. \[\begin{align*}
   y &> -2 \\
   x &< 1
\end{align*}\]

12. \[\begin{align*}
   y &> -x \\
   x + y &\leq 2
\end{align*}\]

Graph the solution set of the system of linear inequalities.

13. \[\begin{align*}
   y &> -2 \\
   x &< 1
\end{align*}\]

14. \[\begin{align*}
   y &> -x \\
   x + y &\leq 2
\end{align*}\]

15. \[\begin{align*}
   y &\geq 2x + 5 \\
   x - \frac{1}{3}y &< 1
\end{align*}\]

16. \[\begin{align*}
   x - 2y &< 6 \\
   3x &\geq 2y - 6
\end{align*}\]

17. WRITING MATH How is the solution set of \(y < 2x + 3\) different from the solution set of \(y \leq 2x + 3\)?

Write a system of linear inequalities for the given graph.

18. \[\begin{align*}
   y &> 3 \\
   x &< 1
\end{align*}\]

19. \[\begin{align*}
   y &< 3 \\
   x &< 1
\end{align*}\]
DATA FILE Use the data on height and weight for men and women on page 650. For Exercises 20–21, use $y$ to represent weight (in pounds) and $x$ to represent height (in inches).

20. Determine an equation for the lower male weights for heights up to 64 in. Determine an equation for the upper male weights for heights up to 64 in. Using the equations, write and graph a system of inequalities by shading the corresponding range of weights.

21. Determine equations for the lower and upper female weight for heights up to 60 in. Write a system of inequalities.

Graph the solution set of the system of linear inequalities.

22. $3 < x + y < 6$
   $x \geq 0$
   $y \geq 0$

23. $2 \leq 2x + y \leq 6$
   $x > 1$
   $y \geq 0$

24. BUDGETING Jasmine needs to earn at least $100 this week. She earns $6 per hour doing gardening and $8 per hour as part-time receptionist. She has only 18 h available to work during the week. Write and graph a system of linear inequalities that models the weekly number of hours Jasmine can work at each job and how much money she needs to earn.

Extended Practice Exercises

25. NUMBER THEORY Find all numbers such that the ordered pairs $(x, y)$ have the following conditions:
   1. $x$ is greater than 1;
   2. $y$ is greater than 0;
   3. the sum of the two numbers is less than 9, and the value of $x + 3y$ is at least 6.
   (Hint: Graph the solution set of the system of inequalities.)

26. Graph the system of inequalities and identify the figure.

   $y \geq -2x + 4$
   $y < -2x + 8$
   $-3 < x - y \leq 3$
   $1 < x \leq 9$

Mixed Review Exercises

Solve. (Lesson 5-1)

27. On one farm, the ratio of brown eggs to white eggs produced by the chickens is 1:3. If 312 eggs are produced, how many are brown?

28. At the amusement park, the ratio of children to adults is 5:2. If 63,000 people visit the park, how many are children?

29. At the mall, the ratio of people buying to people just looking is 8:7. If 9000 people come to the mall, how many will buy something?

30. In Seattle, WA, the ratio of rainy days to non-rainy days is approximately 2:3. In 365 days, how many days will it rain in Seattle?
Review and Practice Your Skills

**Practice Lesson 6-7**

Use determinants to solve each system of equations. Check your answers.

1. \(4x - y = 9\)
   \(x - 3y = 16\)
2. \(3x - 5y = -23\)
   \(5x + 4y = 11\)
3. \(x + 4y = 13\)
   \(5x - 7y = -16\)
4. \(-2x + 3y = 10\)
   \(3x - 5y = 14\)
5. \(-x - y = -15\)
   \(2x - y = 6\)
6. \(2x - 7y = -18\)
   \(x - 2y = -6\)
7. \(3x + y = -1\)
   \(2x - 3y = -8\)
8. \(2y + 3 = 9x\)
   \(3x - y = 6\)
9. \(x - 7 = 3y\)
   \(2(3y + 7) = 5x\)
10. \(3x - 7y = 2\)
    \(6x - 4 = 13y\)
11. \(9x = 6y\)
    \(3x - 4y = -18\)
12. \(x - 6y = 3\)
    \(x + 2y = 5\)
13. \(4x + 6y = 16\)
    \(x = 2y + 1.2\)
14. \(-2x + 3y = 15\)
    \(2x - 3y = 6\)
15. \(4y = 10 - 5x\)
    \(6x + 22 = 2y\)

16. Nadine has $2.35 in dimes and quarters. She has 6 fewer quarters than dimes. How many of each type of coin does she have?

**Practice Lesson 6-8**

Determine whether the given ordered pair is a solution of the given system of inequalities.

17. \((-2, 3); 2x + y < 4\)
   \(-2x + y > 2\)
18. \((-2, 3); x + 3y < 3\)
   \(x + 3y > 3\)
19. \((-2, 3); x < 2\)
   \(y > 3\)

Graph the solution set of the system of linear inequalities.

20. \(x + 2y \leq 3\)
    \(2x - y \leq 1\)
21. \(x \geq -1\)
    \(y > -2\)
22. \(y > 2x - 4\)
    \(y \geq -x - 1\)
23. \(x + y > 2\)
    \(x - y < -2\)
24. \(2x - 3y \leq 9\)
    \(x + 2y < 6\)
25. \(y > 2x\)
    \(y - x \leq 5\)
26. \(x + y < -1\)
    \(3x - y > 4\)
27. \(5x + 2y \geq 12\)
    \(2x + 3y \leq 10\)
28. \(y > -3\)
    \(y < 2x + 4\)
29. \(y < -3x + 2\)
    \(3y \geq x + 15\)
30. \(y > x\)
    \(x \leq -3\)
    \(-x - 3y > 3\)
31. \(2y - x \geq 0\)
    \(x + 5y < 15\)
    \(y \geq x + 1\)
32. \(0.1x + 0.4y \geq -0.8\)
    \(4x - 8y \geq -24\)
33. \(y < -2x + 2\)
    \(6x - 3y \leq -6\)
34. \(0.5x - y < 1\)
    \(x - 2y \geq -6\)
### Practice  Lesson 6-1–Lesson 6-8

Find the slope and \( y \)-intercept for each line. (Lesson 6-1)

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>35. ( y = \frac{5}{7}x - 11 )</td>
<td>36. ( 5x - 7y = 77 )</td>
<td>37. ( 154 + 10x = 14y )</td>
</tr>
<tr>
<td>38. ( y = -3x )</td>
<td>39. ( 6x + 2y = 26 )</td>
<td>40. ( 0.012x + 0.0004y = 0.096 )</td>
</tr>
</tbody>
</table>

Determine whether each pair of lines is parallel, perpendicular, or neither. (Lesson 6-2)

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>41. ( y = 3.2x - 64 ) ( 16x - 5y = 30 )</td>
<td>42. ( y = -4x ) ( y = 4x )</td>
<td>43. ( -7x + 2y = 0 ) ( 3.5y = -x )</td>
</tr>
</tbody>
</table>

Write an equation for the line with the given information. (Lesson 6-3)

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>44. ( m = -\frac{1}{8}, Q(-8, 3) )</td>
<td>45. ( m = \frac{2}{3}, K(12, -5) )</td>
<td>46. ( m = 7.5, b = -2 )</td>
</tr>
<tr>
<td>47. ( G(10, 3), b = 7 )</td>
<td>48. ( P(-6, 2) ) ( Q(6, -2) )</td>
<td>49. ( T(-3, 3.5) ) ( U(2, 16) )</td>
</tr>
</tbody>
</table>

Solve each system of equations by graphing. (Lesson 6-4)

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>50. ( x + y = -1 ) ( y = x + 5 )</td>
<td>51. ( x + y = 5 ) ( y = 2x - 10 )</td>
<td>52. ( 3x + 2y = 13 ) ( 2x - 3y = 26 )</td>
</tr>
<tr>
<td>53. ( x + 4y = -2 ) ( y = -0.25x - 0.5 )</td>
<td>54. ( 4x - 2y = 18 ) ( y = 4x - 12 )</td>
<td>55. ( 3x - 6y = 6 ) ( -x = -2y + 15 )</td>
</tr>
</tbody>
</table>

Solve and check each system of equations by the substitution method. (Lesson 6-5)

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>56. ( y = 2x ) ( x + y = -9 )</td>
<td>57. ( n = 2m - 6 ) ( 2m + n = 10 )</td>
<td>58. ( y = 2x + 5 ) ( x - 2y = 8 )</td>
</tr>
<tr>
<td>59. ( r + 3s = -5 ) ( 3r + 2s = -15 )</td>
<td>60. ( 4x = y + 3 ) ( 9 = 12x - 3y )</td>
<td>61. ( 3a + 4b = -12 ) ( 2b - a = 14 )</td>
</tr>
</tbody>
</table>

Solve each system of equations. Check the solutions. (Lesson 6-6)

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>62. ( 2x + y = 8 ) ( -2x - 3y = -24 )</td>
<td>63. ( 4x + 3y = 8 ) ( 4x - 3y = 32 )</td>
<td>64. ( 2x + y = 4 ) ( 2x + 3y = 24 )</td>
</tr>
<tr>
<td>65. ( 2x + y = -8 ) ( 0.1x + 0.2y = -1.0 )</td>
<td>66. ( 2x - y = 7 ) ( 0.03x + 0.20y = 0.75 )</td>
<td>67. ( 6x + 3y = 0 ) ( -4y = 2x + 12 )</td>
</tr>
</tbody>
</table>

Use determinants to solve each system of equations. Check your answers. (Lesson 6-7)

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>68. ( x + 3y = 11 ) ( 2x - 3y = 13 )</td>
<td>69. ( 5x - y = 16 ) ( 5x + 2y = 13 )</td>
<td>70. ( x = 4y ) ( 4x + 2y = -36 )</td>
</tr>
<tr>
<td>71. ( y = 7x - 1 ) ( 42x - 7y = 49 )</td>
<td>72. ( 2y - 3x = -10 ) ( 2x - y = 6 )</td>
<td>73. ( 0.2x + 0.2y = 0.6 ) ( -9x + 3y = -3 )</td>
</tr>
</tbody>
</table>

Graph the solution set of the system of linear inequalities. (Lesson 6-8)

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>74. ( 2x + y &lt; 7 ) ( y \geq 2(1 - x) )</td>
<td>75. ( x &gt; -2 ) ( y \leq 3 )</td>
<td>76. ( y &gt; -\frac{2}{3}x + 7 ) ( 2x + 3y \leq 9 )</td>
</tr>
</tbody>
</table>
Work with a partner.

1. On graph paper, find the region defined by the following inequalities.
   \[ x \geq 0 \quad y \geq 0 \quad y \leq 5 \quad y + x \leq 10 \]

2. On the same coordinate axes, draw the line \( y = -\frac{1}{2}x \).

3. Place a pencil on its side over the line \( y = -\frac{1}{2}x \). Slowly slide the pencil over the polygonal region keeping it parallel to line \( y = -\frac{1}{2}x \). Name the coordinates of the last point in the region that the pencil passes over.

4. Place the pencil on its side anywhere outside of the polygonal region but not parallel to any of its sides. Slowly slide the pencil over the region. What are the coordinates of the last point in the polygonal region that the pencil passes over?

5. What conclusion can you draw about the last point in the polygonal region that the pencil passes over?

**BUILD UNDERSTANDING**

**Linear Programming** is a method used by business and government to help manage resources and time. Limits to available resources are called constraints. In linear programming, such constraints are represented by inequalities. The intersection of the graphs of a system of constraints is known as a feasible region. The feasible region includes all the possible solutions to the system.

In the activity above, you determined that the last point in the polygonal region that the pencil passed over was located at a vertex. The line represented by the pencil is known as the objective function. The equation of this line can represent quantities such as revenue, profit or cost. In business, the objective function is used to determine how to make the maximum profit with minimum cost.

**Example 1**  

**MANUFACTURING** High Tops Corporation makes two types of athletic shoes: running shoes and basketball shoes. The shoes are assembled by machine and then finished by hand. It takes 0.25 h for the machine assembly and 0.1 h by hand to make a running shoe. It takes 0.15 h on the machine and 0.2 h by hand to make the basketball shoe. At their manufacturing plant, the company can allocate no more than 900 machine hours and 500 hand hours per day. The profit is $10 on each type of running shoe and $15 on each basketball shoe. How many of each type of shoe should be made to maximize the profit?
Solution

If \( x \) represents the number of running shoes made and \( y \) represents the number of basketball shoes made, then the profit objective function \((P)\) is \( P = 10x + 15y \).

We can write inequalities to represent each constraint.

Machine hours: \( 0.25x + 0.15y \leq 900 \)

Hand hours: \( 0.1x + 0.2y \leq 500 \)

To make sure the feasible region is completely within the first quadrant of the coordinate plane, we include the constraints \( x \geq 0 \) and \( y \geq 0 \). Graph the system of inequalities.

The vertices of the feasible region are at \((0, 0)\), \((3600, 0)\), \((0, 2500)\) and \((3000, 1000)\). Evaluate the objective function at each of the vertices of the feasible region.

<table>
<thead>
<tr>
<th>Vertex</th>
<th>( 10x + 15y )</th>
<th>Profit ( P ), dollars</th>
</tr>
</thead>
<tbody>
<tr>
<td>((0, 0))</td>
<td>(10(0) + 15(0))</td>
<td>0</td>
</tr>
<tr>
<td>((3600, 0))</td>
<td>(10(3600) + 15(0))</td>
<td>36,000</td>
</tr>
<tr>
<td>((0, 2500))</td>
<td>(10(0) + 15(2500))</td>
<td>37,500</td>
</tr>
<tr>
<td>((3000, 1000))</td>
<td>(10(3000) + 15(1000))</td>
<td>45,000 (maximum)</td>
</tr>
</tbody>
</table>

Under the given daily constraints, the maximum daily profit the shoe company should expect to make is $45,000. To do this, they would have to produce and sell 3000 running shoes and 1000 basketball shoes per day.

Example 2

Graphing

Using a graphing calculator, graph the solution set of the system of inequalities below to determine the maximum value of \( P = 6x + 2y \).

\[ x + y \leq 3 \quad x \geq 0 \]
\[ y \geq -x + 1 \quad y \geq 0 \]

Solution

Graph the equation that corresponds to each inequality. Make a table of the constraints, related boundary equations, and shading.

<table>
<thead>
<tr>
<th>Inequality</th>
<th>( x + y \leq 3 )</th>
<th>( y \geq -x + 1 )</th>
<th>( x \geq 0 )</th>
<th>( y \geq 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boundary equation</td>
<td>( x + y = 3 )</td>
<td>( y = -x + 1 )</td>
<td>( x = 0 )</td>
<td>( y = 0 )</td>
</tr>
<tr>
<td>Shading</td>
<td>below</td>
<td>above</td>
<td>right of ( y )-axis</td>
<td>above ( x )-axis</td>
</tr>
<tr>
<td>Line</td>
<td>solid</td>
<td>solid</td>
<td>solid</td>
<td>solid</td>
</tr>
</tbody>
</table>

Then locate the vertices of the feasible region using the trace, zoom, and intersect features to determine the coordinates. Make a table of the vertices and the value of \( P \) for each vertex.

The maximum value of \( P \) is 18 when \( x = 3 \) and \( y = 0 \).
TRY THESE EXERCISES

Determine if each point is within the feasible region for \( x \geq 0, y \geq 0, \) and \( 5x + 2y \leq 30. \)

1. \((10, 7)\)  
2. \((1, 6)\)  
3. \((2.75, 6.1)\)  
4. \((5, 2.5)\)

Determine the maximum value of \( P = 15x + 12y \) for each feasible region.

5. \( \)  
6. \( \)

Find the feasible region for each system of constraints. Determine the maximum or minimum value of \( P \) as directed.

7. \(-x + y \geq -2 \quad x \geq 0 \quad y \leq 3\)
8. \(x + y \leq 2 \quad x \geq 0 \quad y \geq 0 \quad x \leq 3\)

maximum, \( P = 5x + y \)
minimum, \( P = 4x + 3y \)

PRACTICE EXERCISES • For Extra Practice, see page 683.

For the feasible region whose vertices are given, find the minimum and maximum value of the objective function and identify the coordinates at which they occur.

9. \( P = 10x + 6y \quad (0, 10) \quad (5, 15) \quad (8, 8) \quad (12, 0)\)
10. \( P = 4x + 5y \quad (2, 5) \quad (2, 9) \quad (6, 11) \quad (8, 5)\)
11. \( P = 1.25x + 0.75y \quad (0, 4) \quad (9, 15) \quad (20, 2)\)
12. \( P = 120x + 180y \quad (6, 6) \quad (6, 10) \quad (10, 12) \quad (13, 11) \quad (13, 6)\)

13. WRITING MATH Why do you think each system of constraints in this lesson contains the inequalities \( x \geq 0 \) and \( y \geq 0 \)? What do these constraints accomplish?

Identify the vertices of the feasible region defined by the constraints.

14. \( x \geq 0; \quad y \geq 0; \quad 7x + 9y \leq 63\)
15. \( x \geq 0; \quad y \geq 0; \quad y + 2x \geq 8\)
16. \( x \geq 0; \quad y \geq 0; \quad y + x \leq 10\)

Determine the minimum value of the objective function \( C = 3x + 2y \) for the graph of each feasible region.

17. \( \)  
18. \( \)
19. **MANUFACTURING** Glimmering Hobbies manufactures remote control cars and airplanes. The plant manufactures at least 50 items but not more than 75 items each week. If the profit is determined by \( P = 25x + 35y \), where \( x \) is the number of cars and \( y \) is the number of planes manufactured, determine the number of cars and airplanes that should be manufactured to maximize the profit.

**SMALL BUSINESS** A group of students are making and selling custom-printed T-shirts and sweatshirts. Their costs are $3.00/T-shirt and $5.00/sweatshirt. A local store owner has agreed to sell their shirts but will only take up to a total of 50 T-shirts and sweatshirts combined. In addition, the store owner said they must sell at least 15 T-shirts and 10 sweatshirts to continue selling at the store. Let \( x \) equal the number of T-shirts and \( y \) equal the number of sweatshirts.

20. The students need to minimize cost. Write the objective function for cost \((C)\).  
21. Write the inequalities that express the constraints.  
22. Graph the inequalities and determine coordinates of the vertices of the feasible region.  
23. How many of each type must they sell to minimize cost?  

**CHAPTER INVESTIGATION** Write a magazine, newspaper, or radio advertisement for your product. Be sure to include a selling price. To establish a selling price, consider manufacturing costs such as materials and labor and the selling price of the other similar products on the market today.

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**EXTENDED PRACTICE EXERCISES**

25. **AGRICULTURE** Valley Farms owns a 3600-acre field. The farmers want to plant Iceberg lettuce which yields $200 per acre and Romaine lettuce which yields $250 per acre. To prevent loss due to disease, the farmers should plant no more than 3000 acres of Iceberg and no more than 2500 acres of Romaine. How many acres of each crop should Valley Farms plant in order to maximize profits? What is the maximum profit?  

26. **SMALL BUSINESS** Sasha owns and operates the Stand-In-Line Skate Shop. She makes a profit of $40 on each pair of adult skates and $20 on each pair of child skates sold. Sasha can stock at most 80 pairs of skates on her shelves. Sasha orders skates once every 6 weeks and can order up to 50 pairs of each type of skate. How many of each type of skate must Sasha stock and sell in a 6-week period in order to maximize her profits? What is the maximum profit?

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**MIXED REVIEW EXERCISES**

Find each product or quotient. (Lesson 1-5)

27. \((-3.9)(-4.8)(-7.6)\)  
28. \(-387 \div [4 \cdot (-3)]\)  
29. \(125 \div (-10) + 38\)  
30. \((8.36)(9.74)(-3.85)\)  
31. \([-4 \cdot (-6)] \div (-2)\)  
32. \((-60) \div [-3 \cdot (-5)]\)
Chapter 6 Review

VOCABULARY

Choose the word from the list at the right that completes each statement.

1. The ___?__ is the ratio of the vertical change to the horizontal change.
2. The graphs of a(n) ___?__ system of linear equations do not intersect.
3. The graphs of a(n) ___?__ system of two linear equations intersect in one point.
4. Two lines are ___?__ if the product of their slopes is $-1$.
5. In the equation $y = 2x - 4$, $-4$ is the ___?__.
6. Two lines are ___?__ if they have the same slope and different $y$-intercepts.
7. The graphs of a(n) ___?__ system of equations are the same line.
8. A(n) ___?__ is a square array of numbers enclosed between two parallel lines.
9. In the ___?__ method for solving a system of equations, a variable in one equation is replaced with an equivalent expression derived from the other equation.
10. The intersection of the graphs of a system of constraints is the ___?__.

LESSON 6-1 ▶ Slope of a Line and Slope-Intercept Form, p. 244

▶ slope $= \frac{\text{rise}}{\text{run}} = \frac{\text{vertical change (change in } y\text{-coordinates)}}{\text{horizontal change (change in } x\text{-coordinates)}}$

▶ A horizontal line has a $0$ slope. The slope of a vertical line is undefined.

▶ The slope-intercept form of an equation of a line is written as $y = mx + b$, where $m$ represents the slope of the line and $b$ represents the $y$-intercept.

11. Find the slope of the line containing the points $A(3, -2)$ and $B(2, 9)$.
12. Graph the line that passes through the point $P(2, -1)$ and has a slope of $\frac{1}{2}$.
13. Graph the line that passes through point $P(8, -6)$ and has a slope of $\frac{-7}{8}$.
14. Find the slope and $y$-intercept for the line with the equation $4y = 2x - 8$. Then graph the equation.
15. Find an equation of the line with slope $=-2$ and $y$-intercept $= 1$.

LESSON 6-2 ▶ Parallel and Perpendicular Lines, p. 248

▶ Two lines with the same slope and different $y$-intercepts are parallel.

▶ Two lines are perpendicular if the product of their slopes is $-1$. 
16. \( MN \) contains the points \( M(4, 6) \) and \( N(-1, 3) \). Find the slope of a line parallel to \( MN \) and the slope of a line perpendicular to \( MN \).

17. Are \( 7x - 3y = 21 \) and \( 7y = 3x + 4 \) parallel, perpendicular, or neither?

18. Determine whether the line containing the points \( T(0, 3) \) and \( U(3, 0) \) is perpendicular or parallel to the line containing the points \( V(7, 1) \) and \( W(1, 7) \).

19. Find the slope of the line parallel to the graph of \( y = -\frac{2}{5}x + 7 \).

20. Find the slope of the line perpendicular to the graph of \( y = 6x - 4 \).

**LESSON 6-3 Write Equations for Lines, p. 254**

- An equation of a line can be written in point-slope form if you know the slope of the line and the coordinates of any point on the line or the coordinates of two points on the line.

21. Write an equation of the line with slope of \(-1\) containing point \( A(2, 3) \).

22. Write an equation of the line containing points \( C(3, 6) \) and \( D(1, -2) \).

23. Write an equation of a line perpendicular to \( y = \frac{1}{2}x - 5 \) containing point \( R(0, -4) \).

24. Write an equation of the line that is perpendicular to \( 4y - 3x = 12 \) containing the point \( Z(2, -1) \).

25. Write an equation of the line whose slope is \(-\frac{1}{4}\) and contains point \( T(8, 2) \).

**LESSON 6-4 Systems of Equations, p. 258**

- Two linear equations with the same two variables form a system of equations. A solution of the system is an ordered pair that makes both equations true. Graphing both equations can be used to solve the system. The point of intersection of the two lines is the solution.

- In an independent system, the graphs intersect in one point. In an inconsistent system, the graphs do not intersect. In a dependent system, the lines coincide.

**Solve each system of equations by graphing.**

26. \[ \begin{align*}
2x + y &= 6 \\
2y + 2 &= 3x
\end{align*} \]

27. \[ \begin{align*}
-6x + 3y &= 18 \\
y &= 6 - 2x
\end{align*} \]

28. \[ \begin{align*}
y &= \frac{1}{2}x \\
2x + y &= 10
\end{align*} \]

29. The perimeter of a rectangle is 40 m. The length is one less than twice its width. What are the dimensions of the rectangle?

**LESSON 6-5 Solve Systems by Substitution, p. 264**

- A system of equations can be solved algebraically. To solve a system using substitution, solve one equation for a variable and then substitute that expression into the second equation.

**Solve each system of equations by the substitution method.**

30. \[ \begin{align*}
y &= 3x \\
x + 2y &= -21
\end{align*} \]

31. \[ \begin{align*}
y &= x + 7 \\
x + y &= 1
\end{align*} \]

32. \[ \begin{align*}
x - 3y &= -9 \\
5x - 2y &= 7
\end{align*} \]

33. Angle \( A \) and \( \angle B \) are supplementary. The measure of \( \angle A \) is 24 degrees greater than the measure of \( \angle B \). Find the measures of \( \angle A \) and \( \angle B \).
LESSON 6-6  Solve Systems by Adding and Multiplying, p. 268

- A system of equations can be solved by adding or subtracting two equations to eliminate one variable. Sometimes, one or both equations must be multiplied by a number or numbers before the equations are added or subtracted.

Solve each system of equations by the substitution method. Check the solutions.

34. \(3x + 2y = -3\)
\(-4x + 2y = 4\)

35. \(2x + 7y = -1\)
\(3x = -2y + 7\)

36. \(3x - 5y = -1\)
\(-5x + 7y = -1\)

37. The concession stand sells hot dogs and soda during Beck High School football games. John bought 6 hot dogs and 4 sodas and paid $10.50. Jessica bought 4 hot dogs and 3 sodas and paid $7.25. What is the cost of one hot dog?

LESSON 6-7  Problem Solving Skills: Determinants & Matrices, p. 274

- Determinants can be used to solve a system of equations.

14. \[
\begin{align*}
ax + by &= e \\
ax + dx &= f 
\end{align*}
\]
\[\det A = ad - bc; \quad x = \frac{ed - bf}{ad - bc}; \quad y = \frac{af - ec}{ad - bc}; \quad ad - bc \neq 0\]

Use determinants to solve each system of equations. Check your answers.

38. \(5y + 3x = 4\)
\(2x - 3y = 5\)

39. \(2y = 4x - 10\)
\(3y + x = -1\)

40. \(4x + 3y = 19\)
\(3x - 4y = 8\)

41. Danielle is 5 years less than twice Mario’s age. In 15 years, Mario will be the same age as Danielle is now. Find the ages of Danielle and Mario in 5 years.

LESSON 6-8  Systems of Inequalities, p. 276

- A system of linear inequalities can be solved graphically. The intersection of the graphs of the inequalities is the solution set of the system.

Graph the solution set of the system of linear inequalities.

42. \(2x + 5y \geq 10\)
\(3x - y \leq 3\)

43. \(2x - 3y \leq 6\)
\(x + y \geq 1\)

44. \(3x + y \geq 2\)
\(y \geq 2x - 1\)

LESSON 6-9  Linear Programming, p 282

- Linear programming can be used to solve business-related linear inequalities.

Determine the maximum value of \(P = 5x + 2y\) for each feasible region.

45.

46.

47.

CHAPTER INVESTIGATION

EXTENSION  Compare your improved product to the original product. Write a report about how your improvements will make the product better. Include an explanation as to how the improvements to the product will justify the increased cost of the product.
Chapter 6 Assessment

Find the slope and $y$-intercept for each line. Then graph the equation.

1. $-3x + 4y = 12$
2. $x + 5y = -5$

Find the slope of each line. Then give the slope of a line parallel to the given line and the slope of a line perpendicular to the given line.

3. The line containing the points $R(1, -1)$ and $S(-4, 1)$.
4. $4x - y = 8$
5. Graph the line that passes through $P(2, -1)$ and has a slope of $-\frac{1}{2}$.

Write an equation of the line with the given information.

6. $\begin{array}{|c|c|c|c|c|}
\hline
\quad & 4 & 2 & \quad & \quad \\
\hline
\quad & 2 & 0 & \quad & \quad \\
\hline
\quad & -2 & -4 & \quad & \quad \\
\hline
\end{array}$

7. $m = -\frac{4}{5}, b = 2$
8. $m = \frac{1}{2}, A(-1, 2)$
9. $C(-4, 1)$ and $D(1, 4)$

Use a graph to solve each system.

10. 8x - 2y = 6
3x = 4y - 1
11. 2y = 3x
3y - 2x = 10
12. x - 2y < 4
3 ≥ x + y
13. y - 3 < 0
x + 2y > 2
x ≤ 2

Solve.

14. $-4x - 5y = -2$
6y = -x + 10
15. $x + 3y = -9$
7 - 2y = 5x
16. $5x - 2y = 9$
x - 4y = 9
17. $-9x - 3y = 9$
x = $\frac{y}{3} + 1$

18. The sum of the digits of a two-digit number is 5. The units digit is one more than 3 times the tens digit. Find the original number.
19. Kari is 6 years older than Adam. In 9 years, $\frac{1}{2}$ of Adam’s age will equal $\frac{1}{3}$ Kari’s age. Find the ages of Kari and Adam in 2 years.
20. One solution contains a 40% acid solution. Another contains a 60% acid solution. Determine the number of liters of each solution needed to make 25 L of a 56% acid solution.
Standardized Test Practice

Part 1  Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. If $A = \{x \mid x \leq 3\}$ and $B = \{x \mid x > -1\}$, what is the least integer in $A \cap B$? (Lesson 1-3)
   - A 3
   - B 0
   - C 1
   - D 2

2. Molly received grades of 79, 92, 68, 90, 72, and 92 on her history tests. What measure of central tendency would give her the highest grade for the term? (Lesson 2-7)
   - A mean
   - B median
   - C mode
   - D range

3. $B$ is the midpoint of $\overline{AC}$. What is the value of $x$ if $AC = 26$ and $AB = 3x - 5$? (Lesson 3-3)
   - A 6
   - B 7
   - C 13
   - D 16

4. In the figure, $\overline{BA} \cong \overline{BC}$ and $\angle A \cong \angle C$. Which postulate could you use to prove $\triangle ABF \cong \triangle CBD$? (Lesson 4-2)
   - A Angle-Angle-Angle Postulate
   - B Angle-Side-Angle Postulate
   - C Side-Angle-Side Postulate
   - D Side-Side-Side Postulate

5. The lengths of two sides of a triangle are 7 in. and 10 in. Which length could not be the measure of the third side? (Lesson 4-6)
   - A 5 in.
   - B 7 in.
   - C 12 in.
   - D 18 in.

6. The area of $\triangle ABC$ is 15 cm$^2$. What is the value of $x$? (Lesson 5-2)
   - A 3 cm
   - B 5 cm
   - C 6 cm
   - D 10 cm

7. Which equation represents a graph that is perpendicular to the graph of $2x + 6y = 8$? (Lesson 6-2)
   - A $3x - 9y = 16$
   - B $12x + 4y = 4$
   - C $5y = 15x + 10$
   - D $21y = 3 - 7x$

8. When solving the following system of equations, which expression could be substituted for $x$? (Lesson 6-5)
   - A $4y - 1$
   - B $1 - 4y$
   - C $3y - 9$
   - D $9 - 3y$

9. Which graph is the solution of the following system of inequalities? (Lesson 6-8)
   - A $y \geq 2x$
   - B $2y + x \leq 3$

Test-Taking Tip

Question 9
If you are allowed to write in the test booklet, cross off each answer choice that you know is not the answer, so you will not consider it again.
Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

10. A developer is going to divide some land for single-family homes. If she buys 12 acres of land, how many \( \frac{3}{4} \)-acre lots can she sell? (Lesson 1-5)

11. Mt. Everest is the highest mountain in the world. It is about \( 2.9 \times 10^4 \) ft above sea level. Write this height in standard form. (Lesson 1-8)

12. What is the next term in the following sequence? (Lesson 2-1)
   \[ 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots \]

13. If \( g(x) = -3x^2 - |x| + 6 \), what is the value of \( g(2) \)? (Lesson 2-3)

14. Solve \(-\frac{5}{6}x + 6 = -9\). (Lesson 2-5)

15. If \( \overline{AC} \perp \overline{DB}, m \angle EBC = (4x + 4)^\circ, m \angle DBE = (3x + 9)^\circ \), find \( m \angle DBE \). (Lesson 3-2)

16. What is the value of \( n \) in the figure below? (Lesson 4-7)

17. What is the surface area of the figure? (Lesson 5-6)

18. Find the height \( x \) if the volume of the rectangular box is 220 cm\(^3\). (Lesson 5-7)

19. What is the slope of \( \overline{MN} \) containing points \( M(-1, 4) \) and \( N(-5, -2) \)? (Lesson 6-1)

20. What is the solution of the system of equations represented by the graph? (Lesson 6-4)

21. If \( 2x + 3y = 9 \) and \( 8x - 5y = 19 \), what is the value of \( 2x \)? (Lesson 6-6)

Part 3 Extended Response

Record your answers on a sheet of paper. Show your work.

22. The vertices of a triangle are \( A(-2, -1), B(-1, 3), \) and \( C(2, 1) \).
   a. Write the equations of the lines that contain the sides of the triangle. (Lesson 6-3)
   b. Write the inequalities that would represent the triangle and its interior. Then, draw the graph. (Lesson 6-9)

23. The manager of a movie theater found that Saturday’s sales were $3675. He knew that a total of 650 tickets were sold Saturday. Adult tickets cost $7.50 each and child tickets cost $4.50 each. (Lessons 6-5, 6-6, and 6-7).
   a. Write a system of equations to represent the situation.
   b. What method would you use to solve the equations? Explain. Solve the equations.