Suppose your family is planning a one-week vacation. If you are like millions of Americans, your plans would probably include a day or two at an amusement park.

The first amusement park in the United States was built in 1895 at Coney Island in New York City. Today, there are hundreds of amusement and theme parks throughout the country. More than 160 million people visit amusement parks across America each year. Many companies specialize in the design and construction of new rides and adventures.

- **Construction Supervisors** (page 347) oversee the construction of new attractions. These workers must pay particular attention to details to assure public safety. Construction supervisors must follow complicated plans, oversee large budgets, and supervise carpenters, electricians, artists, and many other workers.

- **Aerospace Engineers** (page 367) design roller coasters. They use their knowledge of aerodynamics, propulsion, stress tolerances, and gravitational forces to design roller coasters that are fast, exciting, and safe to enjoy.
Use the table for Questions 1–4.

1. Find the average speed in feet per second of Mean Streak and The Beast. Which coaster has the fastest average speed? *(Hint: Use the formula $d = rt$, where $d =$ distance, $r =$ rate, and $t =$ time.)*

2. Some say that the characteristic that most influences the top speed of a coaster is the angle of descent of the first hill. The method for measuring the angle of descent is shown in the diagram at the right. Do you agree with this thinking? Explain your reasoning.

3. How many miles longer is The Beast than The Rattler?

4. If Texas Giant could maintain its top speed for the entire length of the track, what would be the duration of the ride?

---

**Data Activity: Classic Wooden Roller Coasters**

<table>
<thead>
<tr>
<th>Coaster names</th>
<th>Top speed</th>
<th>Height</th>
<th>Track length</th>
<th>Ride duration</th>
<th>Angle of descent of first hill</th>
<th>Vertical drop</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Rattler</td>
<td>55 mi/h</td>
<td>179.6 ft</td>
<td>5080 ft</td>
<td>2:15</td>
<td>61.4°</td>
<td>124 ft</td>
</tr>
<tr>
<td>Shivering Timbers</td>
<td>65 mi/h</td>
<td>125 ft</td>
<td>5384 ft</td>
<td>2:30</td>
<td>53.25°</td>
<td>120 ft</td>
</tr>
<tr>
<td>Texas Giant</td>
<td>65 mi/h</td>
<td>143 ft</td>
<td>4920 ft</td>
<td>2:30</td>
<td>53°</td>
<td>137 ft</td>
</tr>
<tr>
<td>Mean Streak</td>
<td>65 mi/h</td>
<td>161 ft</td>
<td>5427 ft</td>
<td>2:45</td>
<td>52°</td>
<td>155 ft</td>
</tr>
<tr>
<td>Georgia Cyclone</td>
<td>50 mi/h</td>
<td>95 ft</td>
<td>2970 ft</td>
<td>1:48</td>
<td>53°</td>
<td>78.5 ft</td>
</tr>
<tr>
<td>The Beast</td>
<td>65 mi/h</td>
<td>135 ft</td>
<td>7400 ft</td>
<td>3:40</td>
<td>45°</td>
<td>141 ft</td>
</tr>
</tbody>
</table>

---

**Chapter Investigation**

Amusement parks are constantly building new rides to attract new and returning customers. Designing new rides requires an understanding of geometry, physics, and construction techniques. Engineers are always looking for safe ways to provide greater thrills.

**Working Together**

Design an “out-and-back” roller coaster with eight hills. Make a scale drawing of the coaster indicating the height of each hill and the angle of descent. Estimate the track length and top speed of your coaster. Use the Chapter Investigation icons to guide your group’s drawing.
The skills on these two pages are ones you have already learned. Stretch your memory and complete the exercises. For additional practice on these and more prerequisite skills, see pages 654-661.

In this chapter, you will be using matrices to solve equations. It is helpful to know how to find the determinant of a matrix.

**Basic Operations with Integers**

To perform basic operations with matrices, you must be able to do basic operations with integers. Recall that when adding integers whose signs are different, you actually subtract their absolute values and use the sign of the number with the greater absolute value for the answer. When multiplying and dividing integers whose signs are different, your answer is negative.

**Perform the indicated operation.**

1. $-6 + 10$
2. $-8(-3)$
3. $48 \div 2$
4. $-13 - 8$
5. $-2 \times 11$
6. $-19(0)$
7. $4 - 21$
8. $-100 - 1$
9. $45 \div (-9)$
10. $37 + 99$
11. $-13(-1)$
12. $-1 - 2$
13. $-56 \div (-8)$
14. $-3 + (-14)$
15. $12 \times (-12)$

**Determinant of a Matrix**

**Example** Find the determinant of this matrix: $\begin{bmatrix} 4 & -2 \\ 3 & -1 \end{bmatrix}$

To find the determinant, $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$ use the formula $ad - bc$.

$$\begin{vmatrix} 4 & -2 \\ 3 & -1 \end{vmatrix} = 4(-1) - 3(-2) = -4 + 6 = 2$$

**Find the determinant of each matrix.**

16. $\begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix}$
17. $\begin{bmatrix} 8 & -7 \\ 3 & 2 \end{bmatrix}$
18. $\begin{bmatrix} -5 & -1 \\ 1 & 4 \end{bmatrix}$
19. $\begin{bmatrix} 9 & 4 \\ -3 & -2 \end{bmatrix}$
20. $\begin{bmatrix} -1 & 6 \\ 8 & -7 \end{bmatrix}$
21. $\begin{bmatrix} 4 & -5 \\ 2 & 6 \end{bmatrix}$
22. $\begin{bmatrix} 8 & 2 \\ -5 & 6 \end{bmatrix}$
23. $\begin{bmatrix} -7 & -3 \\ -2 & -5 \end{bmatrix}$
24. $\begin{bmatrix} 3 & 0 \\ 4 & 2 \end{bmatrix}$
25. $\begin{bmatrix} 0 & 5 \\ -6 & 2 \end{bmatrix}$
26. $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$
27. $\begin{bmatrix} 0 & 3 \\ 2 & 0 \end{bmatrix}$
Symmetry

A line of symmetry is a line along which a figure can be folded so that the two parts match exactly.

Copy each figure on a sheet of paper. Sketch all the possible lines of symmetry for each figure.

28.

29.

30.

31.

32.

33.

Midpoint Formula

When you are working with figures in the coordinate plane, you may find the midpoint formula is often useful.

Example Find the midpoint of the \( \overline{AB} \) when \( A = (1, 6) \) and \( B = (9, 2) \).

Remember to solve for both coordinates of the midpoint:

\[
\begin{align*}
\text{x(midpoint)} &= \frac{x_1 + x_2}{2} \\
\text{y(midpoint)} &= \frac{y_1 + y_2}{2} \\
x_m &= \frac{1 + 9}{2} \\
y_m &= \frac{6 + 2}{2}
\end{align*}
\]

The coordinates of the midpoint of \( \overline{AB} \) are \( (5, 4) \).

Find the midpoint of each segment that has the given endpoints.

34. \( C(0, 3) \) and \( D(-4, 9) \)  
35. \( E(2, 5) \) and \( F(8, -1) \)

36. \( G(3, 3) \) and \( H(7, 7) \)  
37. \( I(1, 3) \) and \( J(-5, 9) \)

38. \( K(4, 3) \) and \( L(-6, -1) \)  
39. \( M(8, 0) \) and \( N(-4, 8) \)

40. \( O(4, -3) \) and \( P(4, -9) \)  
41. \( Q(6, -2) \) and \( R(-2, 6) \)
Work with a partner. You will need a piece of graph paper, a ruler, and scissors.

1. Using the ruler, draw an isosceles triangle near one bottom corner of your graph paper. Make the base $AB$ 6 units long and the height 4 units. Cut out the triangle and label the vertices $A$, $B$, and $C$.

2. Draw a coordinate plane and label each axis from $-10$ to $10$.

3. Place the triangle in the second quadrant so that $AB$ is parallel to the x-axis. Situate each vertex at the intersection of a horizontal and a vertical line in such a way that $C$ is above $AB$. Trace the triangle and label each vertex to match the original triangle. Label this figure “Triangle 1.”

4. Slide your triangle 9 units straight to the right. Trace the triangle in this new position and label each vertex. Label this figure “Triangle 2.”

5. Turn your triangle so vertex $C$ is below $AB$. Now place the triangle in the fourth quadrant with $AB$ parallel to the x-axis. Trace this triangle. Label this figure “Triangle 3.”

6. Record the slopes of the sides of the three triangles in a table like the one shown.

7. Compare the slopes of the sides of Triangles 1 and 2, Triangles 2 and 3, and Triangles 1 and 3. What do you notice?

### Build Understanding

A **translation** is a *slide* of a figure. It produces a new figure exactly like the original. The new figure is the **image** of the original figure, and the original figure is the **preimage** of the new figure. A translation is an example of a **transformation** of a figure.

Another kind of transformation that yields a congruent figure is a **reflection**, or *flip*. Under a reflection, a figure is reflected, or flipped, over a **line of reflection**.

If a line can be drawn through a geometric figure so that the part of the figure on one side of the line is the reflection of the part of the figure on the opposite side, the figure is said to exhibit line symmetry and the line is a **line of symmetry**, or *axis of symmetry*. A reflection image and its preimage combined will always be a figure that has line symmetry.

### Example 1

Graph the image of parallelogram $MNOP$ with vertices $M(2, 1)$, $N(4, 7)$, $O(7, 7)$, and $P(5, 1)$ under each transformation from the original position.

a. 9 units down

b. reflected across the $y$-axis
Solution

a. To move the given figure 9 units down, subtract 9 from the y-coordinate of each vertex.

\[ M(2, 1) \rightarrow M'(2, 1 - 9) = M'(2, -8) \]
\[ N(4, 7) \rightarrow N'(4, 7 - 9) = N'(4, -2) \]
\[ O(7, 7) \rightarrow O'(7, 7 - 9) = O'(7, -2) \]
\[ P(5, 1) \rightarrow P'(5, 1 - 9) = P'(5, -8) \]

b. The reflection of the point \((x, y)\) across the y-axis is the point \((-x, y)\).

\[ M(2, 1) \rightarrow M''(-2, 1) \]
\[ N(4, 7) \rightarrow N''(-4, 7) \]
\[ O(7, 7) \rightarrow O''(-7, 7) \]
\[ P(5, 1) \rightarrow P''(-5, 1) \]

Example 2

Compare the slopes of corresponding non-horizontal sides for the preimage and each transformation image in Example 1.

Solution

For the first transformation, compare the slopes of \(\overline{MN}\) and \(\overline{M'N'}\), as well as the slopes of \(\overline{OP}\) and \(\overline{O'P'}\). For the second transformation, compare the slopes of \(\overline{MN}\) and \(\overline{M''N''}\), as well as the slopes of \(\overline{OP}\) and \(\overline{O''P''}\).

<table>
<thead>
<tr>
<th>Side</th>
<th>(\overline{MN})</th>
<th>(\overline{M'N'})</th>
<th>(\overline{OP})</th>
<th>(\overline{O'P'})</th>
<th>(\overline{M''N''})</th>
<th>(\overline{O''P''})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>-3</td>
<td>-3</td>
</tr>
</tbody>
</table>

For the translation in part a, corresponding sides have equal slopes. For the reflection in part b, corresponding sides have opposite slopes.

Example 3

RECREATION At a miniature golf course, a hole is designed so that the ball must travel along a line of reflection between two congruent triangular blocks.

Graph the image of \(\triangle ABC\) with vertices \(A(3, 5)\), \(B(5, 8)\), and \(C(1, 7)\) under a reflection across the line whose equation is \(y = x\). Compare the slopes of the corresponding sides of \(\triangle ABC\) and \(\triangle A'B'C'\).

Solution

Graph the line \(y = x\). Graph the reflection image as directed. Use the rule \((x, y) \rightarrow (y, x)\). Make a table to compare the slopes of the corresponding sides. The slopes of corresponding sides are reciprocals of each other.

<table>
<thead>
<tr>
<th>Side</th>
<th>(\overline{AB})</th>
<th>(\overline{BC})</th>
<th>(\overline{CA})</th>
<th>(\overline{A'B'})</th>
<th>(\overline{B'C'})</th>
<th>(\overline{C'A'})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope</td>
<td>(\frac{3}{2})</td>
<td>(\frac{1}{4})</td>
<td>-1</td>
<td>(\frac{2}{3})</td>
<td>4</td>
<td>-1</td>
</tr>
</tbody>
</table>
If an object is symmetrical with respect to a line and you only have half of it, you can draw the other half.

**Example 4**

**ART** The figure shown at the right is half of a figure that has line symmetry. Complete the figure.

**Solution**

Draw a reflection across the line of symmetry for the half of the figure shown.

**TRY THESE EXERCISES**

Copy parallelogram $DEFG$ at the right on a coordinate plane. Then graph its image under each transformation from the original position.

1. 7 units down ($D'E'F'G'$)
2. reflected across the $y$-axis ($D''E''F''G''$)
3. Copy and complete the chart below.

<table>
<thead>
<tr>
<th>Side</th>
<th>$DE$</th>
<th>$D'E'$</th>
<th>$D''E''$</th>
<th>$EF$</th>
<th>$E'F'$</th>
<th>$E''F''$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. Copy $\triangle XYZ$ at the right on a coordinate plane and graph its image under a reflection across the line with equation $y = -x$. Compare the slopes of $XY, X'Y', YZ, Y'Z', XZ$, and $X'Z'$.

5. **WRITING MATH** How can you recognize a line of symmetry?

**PRACTICE EXERCISES** • For Extra Practice, see page 686.

On a coordinate plane, graph parallelogram $HIJK$ with vertices $H(1, 1), I(5, 1), J(8, 4)$, and $K(4, 4)$. Then graph its image under each transformation from the original position.

6. 10 units left
7. reflected across the $x$-axis
8. Compare the slopes of the non-horizontal sides of parallelogram $HIJK$ in all three positions above.
9. Graph the image of $\triangle ABC$ with vertices $A(1, 4), B(5, 6)$, and $C(2, 7)$ under a reflection across the line with equation $y = x$. Compare the slopes of the sides of $\triangle ABC$ and $\triangle A'B'C'$.

10. **YOU MAKE THE CALL** Jenna says that the rule $(x, y) \rightarrow (4-x, y)$ can be used to translate an image 4 units to the left. Do you agree with Jenna's thinking?
Copy each figure below on graph paper along with its line of symmetry. Then complete the figure.

11. 12.


15. **AMUSEMENT PARK DESIGN** A new roller coaster has two entrances to the boarding platform. The eastern entrance is a reflection of the western entrance shown at the right. Copy the entrance on graph paper. Then draw its reflection.

**EXTENDED PRACTICE EXERCISES**

16. Triangle $D'E'F'$ is the image of a figure that was translated under the rule $(x, y) \rightarrow (x + 3, y - 2)$. What are the vertices of the preimage of $\triangle D'E'F'$? What are the slopes of the sides $D'E'$, $E'F'$, and $D'F'$? Are the slopes of the corresponding sides of the preimage the same?

17. How do the slopes of a segment and a translation image compare?

18. **WRITING MATH** Consider a nonvertical, nonhorizontal segment and its reflection image across each of the lines. How do the slopes of the image and preimage compare? Support your answer with an example for each.
   a. $x$-axis
   b. $y$-axis
   c. $y = x$

**MIXED REVIEW EXERCISES**

Solve each proportion and find the value of $x$. Round to the nearest hundredth if necessary. (Lesson 7-1)

19. $\frac{2}{5} = \frac{6}{x}$

20. $\frac{3}{8} = \frac{x}{24}$

21. $\frac{75}{90} = \frac{15}{x}$

22. $\frac{23}{48} = \frac{x}{60}$

23. $\frac{2x}{10} = \frac{6}{15}$

24. $\frac{x - 4}{12} = \frac{x}{8}$

25. $\frac{3}{x + 2} = \frac{18}{36}$

26. $\frac{1}{3} = \frac{5x}{45}$

27. $\frac{72}{48} = \frac{x + 8}{4}$

28. $\frac{125}{3x} = \frac{5}{3}$

29. $\frac{169}{286} = \frac{13}{4x + 2}$

30. $\frac{x - 2}{54} = \frac{8}{48}$

Exercises 31–34 refer to the figure at the right. (Lesson 3-3)

31. Name the midpoint of $HJ$.

32. Name the segment whose midpoint is $B$.

33. Name all the segments whose midpoint is $E$.

34. Assume $Z$ is the midpoint of $BE$. What is its coordinate?
Think about how gears mesh and turn one another. A gear is a mechanical device that transfers rotating motion and power from one part of a machine to another.

1. From the side at which you see the gears, would you say the larger gear turns clockwise or counterclockwise?
2. What fractional part of a turn does it take for a gear tooth to get from the top to a horizontal position?
3. How many degrees does a gear tooth travel from the top to the bottom of the gear?

Another transformation that produces a figure congruent to the original is a rotation, or turn. A figure is rotated, or turned, about a point.

Rotation is described by three pieces of information:

- the point about which the figure is rotated, or the center of rotation.
- the amount of turn expressed as a fractional part of a whole turn, or as an angle of rotation in degrees.
- the rotation direction—clockwise or counterclockwise.

When you rotate a point $180^\circ$ clockwise about the origin, both the $x$-coordinate and the $y$-coordinate are transformed into their opposites.

Example 1

Graph $\triangle QRS$ and its image after a $180^\circ$ clockwise rotation about the origin. Then compare the slopes of $QR$, $Q'R'$, $QS$, and $Q'S'$.

Solution

Use the rule $(x, y) \rightarrow (-x, -y)$. 

$Q(3, 4) \rightarrow Q'(-3, -4)$

$R(1, 1) \rightarrow R'(-1, -1)$

$S(5, 1) \rightarrow S'(-5, -1)$

<table>
<thead>
<tr>
<th>Side</th>
<th>$\overline{QR}$</th>
<th>$\overline{QS}$</th>
<th>$\overline{Q'R'}$</th>
<th>$\overline{Q'S'}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope</td>
<td>$\frac{3}{2}$</td>
<td>$\frac{3}{2}$</td>
<td>$-\frac{3}{2}$</td>
<td>$-\frac{3}{2}$</td>
</tr>
</tbody>
</table>

The slopes of corresponding segments are the same.
When you rotate a figure 90° counterclockwise, the $y$-coordinate is multiplied by $-1$, and then the $x$-coordinate and $y$-coordinate are transposed. That is, $(x, y) \rightarrow (-y, x)$.

**Example 2**

**RIDE MANAGEMENT** Computers are used to signal ride operators when it is safe to begin a new ride cycle. The ride can start when the screen shows a raised flag. A lowered flag tells the operator to wait. On the computer screen, the raised flag contains the points $A(0, 0)$, $B(-2, 2)$, $C(-5, 5)$, and $D(-5, 2)$. Graph the flag and its image after a 90° counterclockwise rotation about the origin. Label the points of the image $A'$, $B'$, $C'$, and $D'$. Then compare the slope of $BC$ with the slope of $B'C'$.

**Solution**

Use the rule $(x, y) \rightarrow (-y, x)$

$A(0, 0) \rightarrow A'(0, 0)$
$B(-2, 2) \rightarrow B'(-2, -2)$
$C(-5, 5) \rightarrow C'(-5, -5)$
$D(-5, 2) \rightarrow D'(-2, -5)$

slope of $BC = \frac{5 - 2}{-5 - (-2)} = \frac{3}{-3} = -1$

slope of $B'C' = \frac{-5 - (-2)}{-5 - (-2)} = \frac{-3}{-3} = 1$

The product of the slopes is $-1$. The lines are perpendicular.

**Example 3**

Triangle $XYZ$ is rotated twice about the origin. Compare the slopes and determine the angle of rotation first for rotation 1 and then for rotation 2.

<table>
<thead>
<tr>
<th>Original position</th>
<th>After rotation 1</th>
<th>After rotation 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Side</td>
<td>Slope</td>
<td>Side</td>
</tr>
<tr>
<td>$XZ$</td>
<td>2</td>
<td>$X'Z'$</td>
</tr>
<tr>
<td>$YZ$</td>
<td>$-\frac{3}{4}$</td>
<td>$Y'Z'$</td>
</tr>
<tr>
<td>$XY$</td>
<td>$\frac{1}{6}$</td>
<td>$X'Y'$</td>
</tr>
</tbody>
</table>

**Solution**

The first rotation is 180° or 360°, because the slopes are equal to the slopes in the original position.

The second rotation is 90° or 270°, because the slopes are the negative reciprocals of the original position slopes. That is, the product of the slopes is $-1$. 

[mathmatters3.com/extra_examples]
TRY THESE EXERCISES

1. Triangle DEF has vertices D(1, 1), E(5, 3), and F(2, 5). Graph \( \triangle DEF \) and its image after a 180° counterclockwise rotation about the origin. Then compare the slopes of the corresponding sides of \( \triangle DEF \) before and after the rotation.

2. **COMPUTER GRAPHICS** A figure contains the points M(0, 0), N(2, -4), O(4, -8), and P(5, -5). Graph the figure and its image after a 90° clockwise rotation about the origin. Use the rule \((x, y) \rightarrow (y, -x)\). Then compare the slopes of NO and \(N'O'\).

3. **ANIMATION** For a television commercial, a triangular logo is animated so that it rotates twice about the origin in a clockwise direction, as shown in the table below. Compare the slopes and determine how much of a rotation was done each time.

<table>
<thead>
<tr>
<th>Original position</th>
<th>After rotation 1</th>
<th>After rotation 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Side</td>
<td>Slope</td>
<td>Side</td>
</tr>
<tr>
<td>(QR)</td>
<td>-1</td>
<td>(QR')</td>
</tr>
<tr>
<td>(RS)</td>
<td>(\frac{3}{5})</td>
<td>(RS')</td>
</tr>
<tr>
<td>(QS)</td>
<td>3</td>
<td>(QS')</td>
</tr>
</tbody>
</table>

4. **WRITING MATH** Describe how translations, reflections, and rotations could be used to create a pattern. Sketch an example.

PRACTICE EXERCISES • For Extra Practice, see page 687.

For each figure, draw the image after the given rotation about the origin. Then calculate the slope of each side before and after the rotation.

5. Use the rule \((x, y) \rightarrow (y, -x)\) for a 90° clockwise rotation.

6. Use the rule \((x, y) \rightarrow (-x, -y)\) for a 180° clockwise rotation.

7. Use the rule \((x, y) \rightarrow (-y, x)\) for a 90° counterclockwise rotation.

8. **ART** To create a pattern, \(\triangle DEF\) is rotated twice about the origin in a clockwise direction, as shown. Compare the slopes to determine how much of a rotation was completed each time.

<table>
<thead>
<tr>
<th>Original position</th>
<th>After rotation 1</th>
<th>After rotation 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Side</td>
<td>Slope</td>
<td>Side</td>
</tr>
<tr>
<td>(DE)</td>
<td>-1</td>
<td>(DE')</td>
</tr>
<tr>
<td>(EF)</td>
<td>(\frac{1}{7})</td>
<td>(EF')</td>
</tr>
<tr>
<td>(DF)</td>
<td>1</td>
<td>(DF')</td>
</tr>
</tbody>
</table>
Use the figure at the right for Exercises 9–13.

9. Which triangle is the rotation image of Triangle 1 about the point \((-3, 0)\)?

10. Which is the translation image of Triangle 1?

11. Which triangle is the reflection image of Triangle 1 across the \(x\)-axis?

12. Which triangle is the reflection image of Triangle 1 across the \(y\)-axis?

13. Which triangle is the rotation image of Triangle 1 180° clockwise about the origin?

14. **CHAPTER INVESTIGATION** An “out-and-back” coaster returns to its starting point when the ride ends. Design an “out-and-back” coaster with eight hills. You may join the hills with lengths of track that wind, dip and turn. Make a scale drawing of your coaster on graph paper.

**Extended Practice Exercises**

15. **WRITING MATH** Make a generalization about corresponding slopes in each situation. Assume that no sides of the triangle are horizontal or vertical.
   a. A triangle is rotated 180° clockwise about the origin.
   b. A triangle is rotated 90° counterclockwise about the origin.
   c. A triangle is rotated 90° clockwise about the origin.

16. If you rotate a figure about its center and it fits back on top of itself in less than a 360° rotation, the figure is said to have **point symmetry**. Point symmetry is a type of rotation symmetry. For example, a square has point symmetry because, when it is rotated 90° about its center, each image vertex falls on top of one of the original vertices. Through how many degrees would you have to rotate a regular hexagon for this to happen? A regular pentagon?

**Mixed Review Exercises**

Find \(x\) in each pair of similar polygons. (Lesson 7-2)

17. \[
\begin{array}{ccc}
4 & \text{ } & 15 \\
\text{x} & \text{ } & 6 \\
\end{array}
\]

18. \[
\begin{array}{ccc}
\text{ } & \text{ } & \\
\text{ } & \text{ } & \\
\end{array}
\]

Determine the slope of the line containing the given pair of points. (Lesson 6-1)

19. \((−2, −3), (−9, −4)\)

20. \((3, 6), (−2, 8)\)

21. \((−4, 5), (3, −1)\)

22. \((4, 8), (−2, −5)\)

23. \((5, −2), (0, −8)\)

24. \((−4, 2), (3, 2)\)

25. \((−1, 6), (5, 2)\)

26. \((4, −2), (−6, 3)\)

27. \((−3, 1), (−3, −5)\)

28. \((−6, −2), (5, −3)\)

29. \((6, −7), (2, −4)\)

30. \((4, 3), (7, 12)\)

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Graph the image of parallelogram $PQRS$ with vertices $P(2, -1)$, $Q(9, -1)$, $R(8, -4)$, and $S(1, -4)$. Then graph its image under each transformation from the original position.

1. 9 units up  
2. reflected across x-axis  
3. reflected across y-axis  
4. Compare the slopes of the non-horizontal sides of parallelogram $PQRS$ with the slopes of the sides of the image.

Graph the image of triangle $DEF$ with vertices $D(1, 7)$, $E(3, 4)$, and $F(5, 11)$. Then graph its image under each transformation from the original position.

5. 15 units down  
6. reflected across y-axis  
7. reflected across $y = x$  
8. Compare the slopes of the sides of triangle $DEF$ with the slopes of the corresponding sides of the three images.

Graph the image of trapezoid $MKLN$ with vertices $M(-2, 3)$, $K(-4, 7)$, $L(-3, 11)$, and $N(0, 11)$. Then graph its image under each transformation from the original position.

9. reflected across y-axis  
10. reflected across $y = x$  
11. reflected across $y = -x$  
12. Compare the slopes of the sides of trapezoid $MKLN$ with the slopes of the corresponding sides of the three images.

Triangle $ABC$ has vertices $A(-3, 1)$, $B(-5, 2)$, and $C(-1, 4)$. Graph the triangle and its image after each of the following rotations about the origin.

13. 90° clockwise  
14. 180° clockwise  
15. 270° clockwise  
16. Reflect the original triangle $ABC$ across the y-axis. Then reflect this new image across the x-axis. To which of the rotations in Exercises 13–15 does this double-reflection correspond?

17. Reflect the original triangle $ABC$ across the line $y = x$. Then reflect this new image across the line $y = -x$. How does this double-reflection compare to the rotations in Exercises 13–15?

18. Triangle $FGH$ is rotated twice about the origin, as shown in the table below. Each rotation angle is between 0° and 360°. Compare the slopes and determine how much the triangle was rotated each time.

<table>
<thead>
<tr>
<th>Original position</th>
<th>After rotation 1</th>
<th>After rotation 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>side</td>
<td>slope</td>
<td>side</td>
</tr>
<tr>
<td>$FG$</td>
<td>$-\frac{2}{3}$</td>
<td>$F'G'$</td>
</tr>
<tr>
<td>$GH$</td>
<td>$\frac{1}{4}$</td>
<td>$G'H'$</td>
</tr>
<tr>
<td>$FH$</td>
<td>$\frac{3}{5}$</td>
<td>$F'H'$</td>
</tr>
</tbody>
</table>
Graph parallelogram $PQRS$ with vertices $P(-1, 2)\), $Q(2, 3)$, $R(4, 7)$, and $S(1, 6)$. Then graph its image under each transformation from the original position. (Lesson 8-1)

19. 12 units to the left  
20. reflected across $y = x$  
21. reflected across $y$-axis

22. Compare the slopes of the sides of parallelogram $PQRS$ with the slopes of the corresponding sides of the three images.

Graph quadrilateral $ABCD$ with vertices $A(5, 4)$, $B(8, -1)$, $C(0, 0)$, and $D(-2, 1)$. Then graph its image under each transformation from the original position. (Lesson 8-1)

23. 5 units to the left, 4 units down  
24. reflected across $x$-axis  
25. reflected across $y$-axis

26. Compare the slopes of the sides of quadrilateral $ABCD$ with the slopes of the corresponding sides of the three images.

Triangle $ABC$ has vertices $A(2, 0)$, $B(8, -2)$, and $C(7, 3)$. Graph the triangle and its image after each of the following rotations about the origin. (Lesson 8-2)

27. 90° clockwise  
28. 180° clockwise  
29. 270° counterclockwise

30. Compare the slopes of the sides of triangle $ABC$ with the slopes of the corresponding sides of the three images.

Amusement parks hire construction workers and supervisors to build new attractions. Whether the attraction is a roller coaster or a carousel, the construction supervisor must adhere to building and safety codes. The supervisor oversees costs, materials, labor, and transportation related to the project. Construction supervisors must be able to follow blueprints and design specifications. For high-speed rides, the supervisor may work closely with an engineer or specialist. When problems arise, the supervisor and the engineer work together to find solutions.

An engineer has designed a coaster with six hills. The final three hills are a reflection of the first three hills. In the diagram below, the first inclines have been straightened.

1. Identify the coordinates of apex of each of the three hills shown.

2. Draw the coaster’s reflection over the $y$-axis. Write the coordinates of the reflections of the points you identified in Exercise 1.
Work with a partner to study the quilt pattern shown.

1. The pattern of a rotated square is used repeatedly in the design. How many different sizes appear?
2. How does the length of a side in the largest pattern compare to the length of a side in the smallest?

A **dilation** is a transformation that produces an image in the same shape as the original figure, but usually of a different size. An image larger than the original figure is called an **enlargement**. An image smaller than the original figure is called a **reduction**. A figure and its dilation image are similar.

The lengths of the sides of a dilation image are obtained by multiplying the lengths of the sides of the original figure by a number called the **scale factor**. If the scale factor is greater than 1, you create an enlargement. If the scale factor is smaller than 1, you create a reduction. The description of a dilation includes the scale factor and the **center of dilation**. The distance from the center of dilation to each point on the image is equal to the distance from the center of dilation to each corresponding point of the original figure times the scale factor.

When the center of dilation is at the origin, you use the following rule to locate points on the dilation image. Let \((x, y)\) represent a point on the original figure, and let \(k\) represent the scale factor.

\[
(x, y) \rightarrow (kx, ky)
\]

**Example 1**

Draw the dilation image of quadrilateral \(MNPQ\) below with vertices at \(M(3, 1)\), \(N(6, 1)\), \(P(6, 3)\), and \(Q(3, 3)\). The center of dilation is the origin, and the scale factor is 2.

**Solution**

Because the center of dilation is at the origin, use the rule \((x, y) \rightarrow (2x, 2y)\).

\[
egin{align*}
M(3, 1) & \rightarrow M'(6, 2) \\
N(6, 1) & \rightarrow N'(12, 2) \\
P(6, 3) & \rightarrow P'(12, 6) \\
Q(3, 3) & \rightarrow Q'(6, 6)
\end{align*}
\]
Sometimes the center of dilation is not at the origin. For example, the center of dilation might be a vertex of the original figure. In this case, the center of dilation and the corresponding vertex of the dilation image are the same.

Example 2

GRAPHIC DESIGN A triangular flag is designed to promote a new attraction at an amusement park. The flags, represented by \(\triangle ABC\) on the grid shown to the right, will also be manufactured in a smaller size. Draw a dilation image of \(\triangle ABC\) with center of dilation at \(A\) and a scale factor of \(\frac{2}{3}\).

Solution

The distance from the center of dilation, \(A\), to \(B\) is 6 units. So the distance from \(A\) to \(B'\) is \(\frac{2}{3} \times 6\), or 4. Count over 4 units from \(A\) to locate \(B'\). The distance from \(A\) to \(C'\) is \(\frac{2}{3} \times 3\), or 2. Count up 2 units from \(A\) to locate \(C'\). Points \(A\) and \(A'\) coincide because \(A\) is the center of dilation.

Try These Exercises

Refer to the figure at the right for Exercises 1–4.

1. What is the image of square \(ABCD\)?
2. What is the center of dilation?
3. How do the lengths of the sides in the image compare to lengths of the sides in square \(ABCD\)?
4. What is the scale factor?

Copy each graph on graph paper. Then draw each dilation image.

5. The center of dilation is the origin and the scale factor is 4. Use the rule \((x, y) \rightarrow (4x, 4y)\).
6. The center of dilation is the origin and the scale factor is \(\frac{1}{2}\).
7. The center of dilation is the origin and the scale factor is 1.5.

8. The center of dilation is point \( A \) and the scale factor is \( \frac{1}{4} \).

9. The center of dilation is point \( M \) and the scale factor is \( \frac{1}{3} \).

10. The center of dilation is point \( S \) and the scale factor is \( \frac{1}{5} \).

The following sets of points are the vertices of figures and their dilation images. For each two sets of points, give the scale factor.

11. \((0, 0), D(0, 4), E(8, 0)\)
    \(C'(0, 0), D'(0, 5), E'(10, 0)\)

12. \((0, 0), T(0, -6), U(-3, -9)\)
    \(S'(0, 0), T'(0, -4), U'(-2, -6)\)

13. \((0, 2), B(2, 2), C(2, -1), D(0, -1)\)
    \(A'(0, 6), B'(6, 6), C'(6, -3), D'(0, -3)\)

14. **BUSINESS** Alice designed a logo for her business. She drew it on a grid, as shown in the diagram below. She wants to create an enlargement that is 5 times as big. Draw the logo on a separate sheet of graph paper. Then draw the enlargement by multiplying all the coordinates by 5.
15. **ART** An artist needs to enlarge the drawing shown at the right so that the resulting figure is twice as large. She uses another method for making enlargements and reductions which does not require a grid. To use this method, pick any convenient point outside the figure. Label the point $O$. Draw line segments from $O$ to various points on the figure. For example, draw a segment from $O$ to point $A$. Then extend the segment from $A$ to point $A'$ so that $OA = AA'$. Continue until you have enough points to complete the drawing.

16. Trace pentagon $ABCDE$ on another sheet. Use the method described in Exercise 15 to make an enlargement of the pentagon 3 times as big. **Hint:** The distance $OA$ will be $\frac{3}{2}$ of the distance from $A$ to $A'$.

**Extended Practice Exercises**

17. **Geometry Software** Draw rectangle $TUVW$ with vertices $T(2, 2), U(2, 6), V(8, 6)$, and $W(8, 2)$ on a coordinate plane using geometric-drawing software. Draw the dilation images of rectangle $TUVW$ with the center of dilation at $T$ and scale factors of 2 and 1/2.

   a. Find the area of rectangle $TUVW$.
   b. Find the area of its enlargement.
   c. Find the area of its reduction.
   d. How does the area of the enlargement compare to the area of the original figure?
   e. How does the area of the reduction compare to the area of the original figure?

18. **Critical Thinking** Describe the effect a scale factor of $-\frac{1}{2}$ would have on a dilation image of rectangle $TUVW$ from Exercise 17.

**Mixed Review Exercises**

Find the actual length of each of the following. (Lesson 7-3)

19. scale length = 6 in. 
   scale is $\frac{1}{2}$ in. : 4 ft
20. scale length = 4.5 cm 
   scale is 1 cm : 8 mi
21. scale length = 5 cm 
   scale is $\frac{1}{2}$ cm : 6 km
22. scale length = 4 in. 
   scale is 1 in. : 12 mi
23. scale length = 9 cm 
   scale is $\frac{1}{2}$ cm : 3 m
24. scale length = 3.75 in. 
   scale is $\frac{1}{4}$ in. : 4 ft
25. scale length = 6.75 cm. 
   scale is 1 cm : 12 m
26. scale length = $3\frac{3}{4}$ in. 
   scale is $\frac{1}{2}$ in. : 3.8 ft
27. scale length = 4.25 cm 
   scale is 1 cm : 24 km

Determine whether each relation is a function. Give the domain and range of each. (Lesson 2-2)

28. $\{(1, 0), (0, -1), (1, -1), (2, 3)\}$
29. $\{(-1, 0), (-2, 1), (1, -1), (0, -2)\}$
30. $\{(-4, 2), (-5, 3), (-6, 4), (-7, 5)\}$
31. $\{(2, 1), (3, 0), (-3, -1), (-2, -2)\}$

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Work with a partner.

Locate the basic pattern outlined in blue in the upper left corner.

1. Could all the other images in the pattern be obtained by using only translations, rotations, or reflections?
2. Describe how you could obtain the pattern outlined in red by applying two successive transformations to the pattern outlined in blue.

**Example 1**

Begin with square $PQRS$ with vertices $P(-5, 2)$, $Q(-1, 2)$, $R(-1, 6)$, and $S(-5, 6)$. First, perform a reflection over the $x$-axis. Then using $P'Q'R'S'$, perform a dilation with center at the origin and a scale factor of 2 to obtain $P''Q''R''S''$.

**Solution**

Start with square $PQRS$. Use the rule $(x, y) \rightarrow (x, -y)$.

- $P(-5, 2) \rightarrow P'(-5, -2)$
- $Q(-1, 2) \rightarrow Q'(-1, -2)$
- $R(-1, 6) \rightarrow R'(-1, -6)$
- $S(-5, 6) \rightarrow S'(-5, -6)$

Then apply a dilation with center at the origin and a scale factor of 2 to square $P'Q'R'S'$. Use the rule $(x, y) \rightarrow (2x, 2y)$.

- $P'(-5, -2) \rightarrow P''(-10, -4)$
- $Q'(-1, -2) \rightarrow Q''(-2, -4)$
- $R'(-1, -6) \rightarrow R''(-2, -12)$
- $S'(-5, -6) \rightarrow S''(-10, -12)$
ENGINEERING  A ride designer is using a computer to map the movement of the car for a new amusement park ride. The two triangles at the right represent the car at the beginning and end of a short section of track after two transformations. Describe these transformations.

Solution

Think of the triangle in Quadrant III as the preimage and the triangle in Quadrant I as the image. Use what you know about transformations to identify the two transformations performed on the preimage.

The preimage has been reflected, or flipped, across the y-axis. The triangle was then moved up, or slid, in a vertical direction for 10 units.

A composite of a reflection followed by a translation (or vice versa) is called a glide reflection.

TRY THESE EXERCISES

1. Perform the following two transformations on rectangle \(JKLM\): a translation five units to the left followed by a reflection over the \(x\)-axis.

2. **WRITING MATH** Begin with rectangle \(CDEF\) at \(C(1, 2), D(1, 4), E(−3, 4),\) and \(F(−3, 2)\). What two transformations could be used to create \(C''D''E''F''\) at \(C''(9, −2), D''(9, −4), E''(5, −4),\) and \(F''(5, −2)\)?

3. Does the order in which you perform the transformations for Exercise 2 affect the final image?

4. Describe two transformations that could be used to create the image in blue.

See additional answers:
ART  To produce a repeating pattern, an artist is asked to perform the following transformations. For Exercises 5–8, draw the result of the first transformation as a dashed figure and the result of the second transformation in red.

5. a translation 8 units to the left, followed by a translation 3 units down.

6. a translation 6 units up, followed by a reflection over the $y$-axis.

7. a reflection over the $y$-axis, followed by a dilation with center at the origin and a scale factor of 2.

8. a clockwise rotation of $180^\circ$ around the origin, followed by a clockwise rotation of $90^\circ$ around the origin.

9. **GRAPHICS DESIGN** A designer has been asked to create a border by repeating a simple pattern that has both vertical and horizontal symmetry. Create the basic pattern using a triangle and multiple transformations. Write a paragraph describing the transformations.

In Exercises 10–11, describe two transformations that would create the image in blue. There may be more than one possible answer.

10.

11.
Tell whether the order in which you perform each pair of transformations affects the image produced. If it does affect the image, sketch an example.

12. a translation followed by another translation
13. a reflection followed by a rotation
14. a rotation followed by another rotation
15. a translation followed by a reflection
16. a reflection followed by another reflection
17. a translation followed by a rotation

**Extended Practice Exercises**

18. In the figure at the right, \( \triangle ABC \) is reflected over line \( p \), and the image is then reflected over line \( q \), where \( p \parallel q \). What kind of single transformation would produce the same result as a composite of reflections over two parallel lines?

19. In the figure at the right, \( \triangle DEF \) is reflected over line \( m \), and then the image is reflected over line \( n \), where lines \( m \) and \( n \) are intersecting lines. What kind of single transformation would produce the same result as a composite of reflections over two intersecting lines?

20. Trace the trapezoid shown at the right on another sheet of paper. Make three other copies of the trapezoid. Cut out the four trapezoids and rearrange them to form a larger trapezoid that is the same shape as the smaller trapezoid.

**Mixed Review Exercises**

Determine whether each pair of triangles is similar. If the triangles are similar, give a reason: write AA, SSS, or SAS. (Lesson 7-4)

21.

22.

23.

24.
PRACTICE  ■  LESSON 8-3

Using the figure on the right, draw each dilation image on a separate coordinate grid.

1. center of dilation is origin; scale factor is 3
2. center of dilation is origin; scale factor is \( \frac{1}{2} \)
3. center of dilation is \( A \); scale factor is 2.5
4. center of dilation is \( B \); scale factor is \( \frac{3}{4} \)

Using the figure on the right, draw each dilation image on a separate coordinate grid.

5. center of dilation is origin; scale factor is 1.5
6. center of dilation is origin; scale factor is \( \frac{1}{4} \)
7. center of dilation is \( F \); scale factor is 2
8. center of dilation is \( G \); scale factor is 3

The following two sets of points are the vertices of triangles and their dilation images. Name the scale factor and the center of dilation for each.

9. \( D(-5, -5), E(10, 0), F(-5, 0) \)  
   \( D'(-5, -1), E'(-2, 0), F'(-5, 0) \)  

10. \( S(2, 2), T(4, 8), U(6, 4) \)  
    \( S'(6, 6), T'(12, 24), U'(18, 12) \)

PRACTICE  ■  LESSON 8-4

For each exercise, draw the result of the first transformation as a dashed figure and the result of the second transformation in red.

11. clockwise rotation of 90°; reflection across \( x \)-axis
12. reflection across \( y = -x \)  
13. dilation—center at origin, scale factor of 2; translation 5 units up

Determine the transformations necessary to create figure 2 from figure 1. There may be more than one possible answer.

14. 
15. 
16.
Graph the image of parallelogram $PQRS$ with vertices $P(5, 0)$, $Q(-1, -1)$, $R(1, 3)$, and $S(7, 4).$ Then graph its image under each transformation from the original position. (Lesson 8-1)

17. 5 units down, 6 units left
18. reflected across $y = x$
19. reflected across $y = -2$
20. Compare the slopes of parallelogram $PQRS$ in all four positions above.

Triangle $ABC$ has vertices $A(-2, 0)$, $B(1, 3)$, and $C(2, 7).$ Graph the triangle and its image after each of the following rotations about the origin. (Lesson 8-2)

21. $90^\circ$ clockwise
22. $180^\circ$ counterclockwise
23. $90^\circ$ counterclockwise
24. Compare the slopes of triangle $ABC$ in all four positions above.

The following sets of points are the vertices of triangles and their dilation images. Name the scale factor and the center of dilation for each. (Lesson 8-3)

25. $D(-5, -3), E(-5, 7), F(-1, 7)$
   $D'(-15, -9), E'(-15, 21), F'(-3, 21)$
26. $D(-5, -3), E(-5, 7), F(-1, 7)$
   $D'(-5, -18), E'(-5, 7), F'(5, 7)$

Mid-Chapter Quiz

On a coordinate grid, graph parallelogram $ABCD$, with vertices $A(1, 6)$, $B(2, 9)$, $C(4, 6)$, and $D(3, 3)$ and the following transformation images. (Lesson 8-1)

1. reflected across $x$-axis
2. translated two units right
3. $\triangle D'E'F'$ is the image after a rotation of $270^\circ$ clockwise about the origin of $\triangle DEF$ with vertices $D(-5, -2), E(-2, -1),$ and $F(-6, -7).$ Find the coordinates of $D', E'$, and $F'$ and the slopes of $DF$ and $D'F'$. (Lesson 8-2)
4. $\triangle ABC$ with vertices $A(1, -3), B(6, -1)$, and $C(4, -5)$ has been rotated to $\triangle A'B'C'$ with vertices $A'(-3, 1), B'(1, 6)$, and $C'(5, 4)$. Name two rotations that could have been used. (Lesson 8-2)
5. Graph parallelogram $DEFG$, with vertices $D(-3, -1), E(1, 3), F(6, 2),$ and $G(2, -2)$, and the dilation image of parallelogram $DEFG$ if the center of dilation is $D$ and the scale factor is $\frac{1}{2}$. (Lesson 8-3)
6. Find the scale factor for $\triangle ABC$, with vertices $A(0, 0), B(3, 6)$, and $C(9, 3)$, and dilation image $A'B'C'$, with vertices $A'(0, 0), B'(4, 8)$, and $C'(12, 4)$. (Lesson 8-3)
7. $\triangle ABC$ has vertices $A(-2, -1), B(-3, -4)$, and $C(-5, -3).$ Perform a reflection across the $y$-axis to obtain $\triangle A'B'C'$ followed by a reflection across the $x$-axis to obtain $\triangle A''B''C''$. (Lesson 8-4)
8. $\triangle D'E''F''$ with vertices $D''(1, 3), E''(6, 3)$ and $F''(6, 1)$ is the image after two transformations of the $\triangle DEF$ with vertices $D(-4, -3), E(1, -3)$, and $F(1, -1)$. Describe the two transformations used to create $\triangle D'E''F''$. (Lesson 8-4)
Addition and Multiplication with Matrices

Goals
- Identify matrices and elements within each matrix by rows and columns.
- Perform addition and scalar multiplication on matrices.

Applications
- Souvenir Sales, Cryptography, Manufacturing

Work with a partner to make a table of the following information.

Alsip and Bell work for Ski Park West. During the winter months, the company rents snowmobiles to park visitors. During December, January, and February, Alsip rented out 18, 12 and 15 snowmobiles, respectively, while Bell rented out 21, 15 and 8 during the same months.

1. Copy and complete the table shown at the right.
2. Make another table with the information, but reverse the rows and columns.

Build Understanding

A matrix (plural: matrices) is a rectangular array of numbers arranged into rows and columns. Usually, square brackets enclose the numbers in a matrix. An array with \( m \) rows and \( n \) columns is called an \( m \times n \) (read \( m \) by \( n \)) matrix.

The dimensions of the matrix are \( m \) and \( n \). The numbers that make up the matrix are the elements of the matrix. The information in the snowmobile problem above can be shown by two different matrices, \( M_1 \) and \( M_2 \). The titles of rows and columns are not part of the matrices.

Matrix \( M_1 \)

<table>
<thead>
<tr>
<th></th>
<th>Alsip</th>
<th>Bell</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dec.</td>
<td>18</td>
<td>21</td>
</tr>
<tr>
<td>Jan.</td>
<td>12</td>
<td>15</td>
</tr>
<tr>
<td>Feb.</td>
<td>15</td>
<td>8</td>
</tr>
</tbody>
</table>

Matrix \( M_2 \)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Alsip</td>
<td>18</td>
<td>12</td>
<td>15</td>
</tr>
<tr>
<td>Bell</td>
<td>21</td>
<td>15</td>
<td>8</td>
</tr>
</tbody>
</table>

Matrix \( M_1 \) has 3 rows and 2 columns. It is a \( 3 \times 2 \) matrix. Matrix \( M_2 \) has 2 rows and 3 columns. So it is a \( 2 \times 3 \) matrix. Although both matrices show the same information, they are not considered to be equal.

Two matrices are equal matrices if and only if they have the same dimensions and corresponding elements are equal.

Example 1

a. Find the dimensions of matrix \( C \).

b. Identify the elements \( C_{32}, C_{21}, \) and \( C_{13} \).

\[
C = \begin{bmatrix}
17 & -2 & 3 \\
0 & -1 & 5 \\
6 & 1 & -7 \\
5 & -4 & 2
\end{bmatrix}
\]
Solution

- **a.** \( C \) has 4 rows and 3 columns. So \( C \) is a \( 4 \times 3 \) matrix.
- **b.** \( C_{32} \) means the element in row 3, column 2.
  \[ C_{32} = 1, \ C_{21} = 0, \ C_{13} = 3 \]

**SOUVENIR SALES** A theme park sells red, white and blue sweatshirts in small, medium and large sizes. A manager at a souvenir stand receives two shipments of sweatshirts.

<table>
<thead>
<tr>
<th>Shipment 1</th>
<th>Shipment 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>red</td>
<td>blue</td>
</tr>
<tr>
<td>Small</td>
<td>18</td>
</tr>
<tr>
<td>Med</td>
<td>15</td>
</tr>
<tr>
<td>Large</td>
<td>16</td>
</tr>
</tbody>
</table>

To find the combined inventory, she adds the corresponding elements in each matrix. This illustrates the concept of matrix addition.

If two matrices \( M \) and \( N \) have the same dimensions, their sum \( M + N \) is the matrix in which each element is the sum of the corresponding elements in \( M \) and \( N \).

**Example 2**

Find the sum of the matrices shown for Shipment 1 and Shipment 2 above.

**Solution**

Call the matrices \( A \) and \( B \).

\[
A = \begin{bmatrix} 18 & 12 & 15 \\ 15 & 33 & 14 \\ 16 & 15 & 18 \end{bmatrix}, \quad B = \begin{bmatrix} 15 & 13 & 12 \\ 18 & 15 & 16 \\ 13 & 8 & 16 \end{bmatrix}
\]

\[
A + B = \begin{bmatrix} 18 + 15 & 12 + 13 & 15 + 12 \\ 15 + 18 & 33 + 15 & 14 + 16 \\ 16 + 13 & 15 + 8 & 18 + 16 \end{bmatrix} = \begin{bmatrix} 33 & 25 & 27 \\ 33 & 48 & 30 \\ 29 & 23 & 34 \end{bmatrix}
\]

Suppose the shop owner wanted to double her total inventory for the holiday season. She could simply double each element of the matrix. This illustrates an operation on matrices called scalar multiplication.

A matrix can be multiplied by a constant \( k \) called a scalar. The product of a scalar \( k \) and matrix \( A \) is the matrix \( kA \) in which each element is \( k \) times the corresponding element in \( A \).

**Example 3**

**CRYPTOGRAPHY** The matrix below represents numerical information that must be transmitted electronically. As the first step in encrypting the information, the matrix is multiplied by 5.

Find the product: 
\[
5 \begin{bmatrix} 8 & 12 & 10 \\ 3 & 4 & -3 \\ -2 & 0 & 6 \end{bmatrix}
\]
SOLUTION

Every element in the matrix must be multiplied by 5.

\[
5 \begin{bmatrix} 8 & 12 & 10 \\ 3 & 4 & -3 \\ -2 & 0 & 6 \end{bmatrix} = \begin{bmatrix} 5 \cdot 8 & 5 \cdot 12 & 5 \cdot 10 \\ 5 \cdot 3 & 5 \cdot 4 & 5 \cdot (-3) \\ 5 \cdot (-2) & 5 \cdot 0 & 5 \cdot 6 \end{bmatrix} = \begin{bmatrix} 40 & 60 & 50 \\ 15 & 20 & -15 \\ -10 & 0 & 30 \end{bmatrix}
\]

**TRY THESE EXERCISES**

1. Find the dimensions of \( M \).

\[
M = \begin{bmatrix} 2 & -1 & 4 & 3 \\ 1 & 0 & -4 & 5 \\ 6 & -1 & 2 & 10 \end{bmatrix}
\]

2. Identify the elements \( M_{34} \) and \( M_{12} \).

3. How many elements are in a \( 5 \times 2 \) matrix?

4. Refer to matrix \( M \) above at the right. Find \( 6M \).

5. Find \( C + D \) if \( C = \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix} \) and \( D = \begin{bmatrix} -3 \\ 1 \\ -2 \end{bmatrix} \).

6. Three schools have the following win–loss record: Appleton, 6 wins, 9 losses; Carrollton, 14 wins, 2 losses; Prestonsville, 12 wins, 5 losses. Show this information in a matrix.

**PRACTICE EXERCISES • For Extra Practice, see page 688.**

Find the dimensions of each matrix.

7. \[
\begin{bmatrix} 2 & -5 & 3 \\ 1 & 4 & -6 \end{bmatrix}
\]

8. \[
\begin{bmatrix} 11 & -8 & 3 & 5 \\ 1 & 0 & 0 & 7 \\ -8 & 0 & 0 & 0 \end{bmatrix}
\]

9. \[
\begin{bmatrix} 0 & 3 \\ 1 & -1 \\ 5 & -5 \\ 2 & 7 \\ 8 & 1.9 \end{bmatrix}
\]

Use the following matrices in Exercises 10–18.

\[
A = \begin{bmatrix} 2 & -5 & 1 \\ 3 & 1 & -7 \end{bmatrix} \quad B = \begin{bmatrix} -12 & 4 & 6 \\ 8 & -2 & -4 \end{bmatrix} \quad C = \begin{bmatrix} 6 & -2 & -3 \\ -4 & 1 & 2 \end{bmatrix}
\]

Find each of the following.

10. \( 2A \)  
11. \( -\frac{1}{2}B \)  
12. \( \frac{3}{2}C \)  
13. \( A + B \)  
14. \( A + C \)  
15. \( A + (-2B) \)  
16. \( B + 2C \)  
17. \( A + \frac{1}{2}B \)  

18. **MANUFACTURING**  In June, a boat manufacturer produced 12 sailboats, 18 catamarans, and 8 yachts. In July, the company produced 10 sailboats, 9 catamarans, and 9 yachts. In August, it produced 9 sailboats, 8 catamarans, and 12 yachts. Write two different \( 3 \times 3 \) matrices to show this information.

19. **DATA FILE**  Use the data on the calorie count of foods on page 650. Create two different matrices showing the amount of the serving and the calorie count for white bread, whole milk, and spaghetti with meatballs.

20. **CHAPTER INVESTIGATION**  Using the scale drawing of your roller coaster, estimate the track length of the entire coaster. Write a paragraph explaining how you arrived at your estimate.
21. **WRITING MATH** Describe a method for remembering the difference between rows and columns.

Matrix subtraction is defined by using the scalar \(-1\).
If \(A\) and \(B\) are matrices with dimensions \(m \times n\), then \(A - B = A + (-1)B\).

**Use the above definition in Exercises 23–26.**

22. \[
\begin{bmatrix}
2 & -2 \\
3 & 7
\end{bmatrix} - \begin{bmatrix}
2 & -2 \\
3 & 7
\end{bmatrix}
\]

23. \[
\begin{bmatrix}
5 & -1 \\
7 & 6
\end{bmatrix} - \begin{bmatrix}
1 & -5 \\
4 & 3
\end{bmatrix}
\]

24. \[
\begin{bmatrix}
0 & 4 & -2 \\
-5 & 1 & 3
\end{bmatrix} - \begin{bmatrix}
6 & -7 & -1 \\
0 & -4 & 8
\end{bmatrix}
\]

25. \[
\begin{bmatrix}
17 & 6 & 12 \\
-3 & -5 & 9
\end{bmatrix}
\]

Remember that two matrices are equal if and only if they have the same dimensions and all their corresponding elements are equal. Use this definition to solve for \(x\) and \(y\).

26. \[
\begin{bmatrix}
x & 4y \\
\end{bmatrix} = \begin{bmatrix}
y + 5 & 2x + 10
\end{bmatrix}
\]

27. \[
\begin{bmatrix}
x + y \\
x - y
\end{bmatrix} = \begin{bmatrix}
9 \\
4
\end{bmatrix}
\]

**Extended Practice Exercises**

28. **YOU MAKE THE CALL** Dawnae says that any two matrices can be added together. Do you agree? If not, why not?

Refer to matrices \(R\) and \(S\) for Exercises 30 and 31.

\[
R = \begin{bmatrix}
4 & 6 \\
3 & 6
\end{bmatrix}, \quad S = \begin{bmatrix}
6 & 5 \\
1 & -3
\end{bmatrix}
\]

29. Find \(R + S\) and \(S + R\). Does addition of matrices seem to be a commutative operation?

30. Find \(R - S\) and \(S - R\). Does subtraction of matrices seem to be a commutative operation?

31. **POPULATION** The matrices \(E\), \(W\), and \(N\), shown at the right, give the enrollments by gender and grade at East, West, and North High Schools. In each matrix, Row 1 gives the number of boys and Row 2 the number of girls. Columns 1 to 4 give the number of students in grades 9 through 12, respectively. Calculate entries in matrix \(T\) that show the total enrollment by gender and grade in the three schools.

\[
E = \begin{bmatrix}
9 & 10 & 11 & 12 \\
180 & 220 & 265 & 250
\end{bmatrix}, \quad W = \begin{bmatrix}
306 & 300 & 340 & 310 \\
290 & 314 & 270 & 350
\end{bmatrix}
\]

\[
N = \begin{bmatrix}
408 & 410 & 406 & 389 \\
380 & 420 & 444 & 370
\end{bmatrix}
\]

32. Find \(x\) to the nearest tenth in the pair of similar triangles. (Lesson 7-5)

Given \(f(x) = -2x - 5\) and \(g(x) = 3x^2\), find each value. (Lesson 2-2)

33. \(f(-3)\) \hspace{1cm} 34. \(f(2)\) \hspace{1cm} 35. \(f(-8)\) \hspace{1cm} 36. \(f(5)\)

37. \(g(-2)\) \hspace{1cm} 38. \(g(3)\) \hspace{1cm} 39. \(g(-5)\) \hspace{1cm} 40. \(g(4)\)
A theme park sells three kinds of tickets. Adults over age 18 pay $15, students from 13 through 18 pay $10, and children under 13 pay $5. On one day, the park sells 280 adult tickets, 420 student tickets, and 382 children's tickets. Notice how this information can be shown by two matrices.

<table>
<thead>
<tr>
<th>number of tickets</th>
<th>cost per ticket</th>
</tr>
</thead>
<tbody>
<tr>
<td>[280 420 382]</td>
<td>[15 10 5]</td>
</tr>
<tr>
<td>adult  student</td>
<td>adult  student</td>
</tr>
<tr>
<td>child</td>
<td>child</td>
</tr>
</tbody>
</table>

Write an expression to show how to compute the total receipts for the day, and then find the total receipts.

**Example 1**

**TICKET SALES** Multiply the two matrices at the top of the page to find the total receipts from ticket sales for the theme park.

**Solution**

The first matrix is a $1 \times 3$ matrix and the second is a $3 \times 1$ matrix. So the product will be a $1 \times 1$ matrix. Use row-by-column multiplication.

$$
\begin{bmatrix}
15 \\
10 \\
5
\end{bmatrix} \cdot 
\begin{bmatrix}
280 \\
420 \\
382
\end{bmatrix} = 280 \cdot 15 + 420 \cdot 10 + 382 \cdot 5
= 4200 + 4200 + 1910
= 10,310
$$

The total receipts equal $10,310.
Example 2

Let \( M = \begin{bmatrix} 1 & 3 & 4 \\ 5 & -2 & 6 \end{bmatrix} \) and \( N = \begin{bmatrix} 1 & 4 & 3 & 0 \\ 5 & -2 & 1 & 6 \\ -4 & 0 & 5 & 7 \end{bmatrix} \). Find the dimensions of \( MN \).

Solution

Because \( M \) is a \( 2 \times 3 \) matrix and \( N \) is a \( 3 \times 4 \) matrix, \( MN \) is a \( 2 \times 4 \) matrix.

\[
\begin{array}{c}
2 \times 3 \\
3 \times 4 \\
2 \times 4
\end{array}
\]

(Notice that you cannot find the product \( NM \), because the number of columns in \( N \) is not the same as the number of rows in \( M \).)

Example 3

**ENCRYPTION** A business uses a coding matrix to encrypt customer account numbers. Matrix \( A \) includes the last four digits of a customer's account number. Matrix \( B \) is the coding matrix.

Let \( A = \begin{bmatrix} 1 & 3 \\ 5 & -2 \end{bmatrix} \) and \( B = \begin{bmatrix} 8 & 7 & 0 \\ 4 & 2 & 6 \end{bmatrix} \). Find \( AB \).

Solution

Because \( A \) is a \( 2 \times 2 \) matrix and \( B \) is a \( 2 \times 3 \) matrix, the product is a \( 2 \times 3 \) matrix. The product of row 1 of \( A \) and column 1 of \( B \) is \( 1(8) + 3(4) = 20 \). Write 20 in row 1 and column 1 of the product matrix.

\[
\begin{bmatrix}
1 & 3 \\
5 & -2
\end{bmatrix}
\begin{bmatrix}
8 & 7 & 0 \\
4 & 2 & 6
\end{bmatrix}
= \begin{bmatrix}
20 & \_ & \_
\end{bmatrix}
\]

The product of row 1 of \( A \) and column 2 of \( B \) is \( 1(7) + 3(2) = 13 \). Write 13 in row 1 and column 2 of the product.

\[
\begin{bmatrix}
1 & 3 \\
5 & -2
\end{bmatrix}
\begin{bmatrix}
8 & 7 & 0 \\
4 & 2 & 6
\end{bmatrix}
= \begin{bmatrix}
20 & 13 & \_
\end{bmatrix}
\]

The other elements in the product are formed by using this row \( \times \) column pattern.

For instance, the element in the second row, third column of the product is found by multiplying the second row of \( A \) by the third column of \( B \). This answer is shown below, along with the final result.

\[
\begin{bmatrix}
1 & 3 \\
5 & -2
\end{bmatrix}
\begin{bmatrix}
8 & 7 & 0 \\
4 & 2 & 6
\end{bmatrix}
= \begin{bmatrix}
20 & 13 & 18 \\
32 & 31 & -12
\end{bmatrix}
\]
TRY THESE EXERCISES

Refer to the matrices below. Find the dimensions of each product, if possible. Do not multiply. If it is not possible to multiply, write NP.

\[ P = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix} \quad Q = \begin{bmatrix} 1 & 6 & 7 \\ 9 \end{bmatrix} \quad R = \begin{bmatrix} 2 & 3 \\ 1 & 4 \\ 6 & 2 \end{bmatrix} \quad S = \begin{bmatrix} 5 \\ 3 \end{bmatrix} \]

1. \( PQ \)  
2. \( PR \)  
3. \( SQ \)  
4. \( RS \)

5. \( QP \)  
6. \( SR \)  
7. \( QS \)  
8. \( SP \)

Find each product, if possible. If not possible, write NP.

\[ A = \begin{bmatrix} 10 & 18 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ 6 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 6 & 1 \\ 5 & 0 \\ 3 & 3 \end{bmatrix} \]

9. \( AB \)  
10. \( AC \)  
11. \( CA \)

PRACTICE EXERCISES • For Extra Practice, see page 689.

Refer to the matrices below. Find the dimensions of each product, if possible. Do not multiply. If it is not possible to multiply, write NP.

\[ D = \begin{bmatrix} 1 & 3 & 5 \\ 0 & 6 & 4 \end{bmatrix} \quad E = \begin{bmatrix} 0 & 4 \\ 5 & 6 \\ 6 & 1 \end{bmatrix} \quad F = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} \quad G = \begin{bmatrix} 2 & 3 & 6 \end{bmatrix} \]

12. \( DE \)  
13. \( ED \)  
14. \( FG \)  
15. \( GF \)

16. \( EG \)  
17. \( FD \)  
18. \( DF \)  
19. \( GE \)

MATRICES Find each product using a graphing calculator. If not possible, write NP.

20. \[ \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \end{bmatrix} \]

21. \[ \begin{bmatrix} 2 & 1 \\ 5 \end{bmatrix} \begin{bmatrix} -2 \\ -3 \end{bmatrix} \]

22. \[ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix} \]

23. \[ \begin{bmatrix} -2 & 4 & 0 \\ -3 & 0 & -8 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \]

Find each product. If not possible, write NP.

24. \[ \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} \]

25. \[ \begin{bmatrix} 3 & 0 & 1 \\ 5 & 0 & 6 \end{bmatrix} \begin{bmatrix} 6 \\ 4 \\ 1 \end{bmatrix} \]

26. \[ \begin{bmatrix} -2 & 4 & 0 \\ -3 & 0 & -8 \end{bmatrix} \begin{bmatrix} -1 & -2 & -3 \\ 0 & 1 & 0 \\ 4 & 5 & 2 \end{bmatrix} \]

27. INVENTORY Find the product \( JK \), which gives the number of small, medium and large T-shirts in inventory at two souvenir stands.

\[ J = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} \quad K = \begin{bmatrix} 3 & 5 & 1 \\ 2 & 1 & 3 \end{bmatrix} \]
28. Use the rule $A(kB) = (kA)B = k(AB)$ to compute: \[
\begin{bmatrix}
2 & 8 \\
4 & 2
\end{bmatrix}
\begin{bmatrix}
\frac{1}{2} & 3 \\
-2 & 1
\end{bmatrix}.
\]
29. **ENCRYPTION** The data in $A$ must be encrypted by multiplying by $B$. Find $AB$ and $BA$.

\[
A = \begin{bmatrix}
2 & 1 \\
4 & 3
\end{bmatrix}, \quad B = \begin{bmatrix}
5 & 1 \\
-3 & 2
\end{bmatrix}
\]

30. **WRITING MATH** What can you conclude about the multiplication of matrices from the products in Exercise 29?

31. For $A = \begin{bmatrix} 3 & 2 \\ -1 & 5 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find $AI$ and $IA$.

32. **WRITING MATH** What can you conjecture about matrix $I$ in Exercise 31?

33. For $A = \begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 2 \\ 1 & 5 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 1 \\ -3 & 2 \end{bmatrix}$ show that $A(BC) = (AB)C$.

### Extended Practice Exercises

34. If $M^2$ means $M \times M$, find the matrix $M^2$ if $M = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$.

35. Find $X^2$, $X^3$, and $X^4$ if $X = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$.

36. For $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, find $A^3$.

### Park Admissions

A theme park offers a discount to members of a travel club. Table A shows the daily ticket sales for the park. Table B shows the average cost per person for park attractions.

<table>
<thead>
<tr>
<th></th>
<th>Club</th>
<th>Non-Club</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adult</td>
<td>2000</td>
<td>1700</td>
</tr>
<tr>
<td>Children</td>
<td>5400</td>
<td>4200</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Admission</th>
<th>Food</th>
<th>Souvenirs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Club</td>
<td>$23</td>
<td>$18</td>
<td>$30</td>
</tr>
<tr>
<td>Non-Club</td>
<td>$26</td>
<td>$20</td>
<td>$35</td>
</tr>
</tbody>
</table>

37. Write matrices for the $A$ and $B$ tables and find the product $AB$ to find the amount spent by park visitors by category.

38. What do the rows and columns of the product matrix $AB$ represent?

39. **CRITICAL THINKING** Solve for $x$ and $y$: \[
\begin{bmatrix} 6 & 2 \\ 8 & 14 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -6 \end{bmatrix}.
\]

### Mixed Review Exercises

Solve each system of equations by graphing. (Lesson 6-4)

40. $y = -3x - 4$
   $y = -2x - 8$

41. $2y = 4x + 10$
   $3y = -3x + 6$

42. $4x = 2y - 6$
   $-3y = x + 5$

43. $5x = y + 7$
   $-4y + x = -10$

44. $-9(x - 3) = 6y$
   $-4y + 28 = 8x$

45. $3y = 2(x + 1.5)$
   $5x = -2(-3y)$

[mathmatters3.com/self_check_quiz]
Use matrices $A$, $B$, and $C$ to find each of the following.

$$A = \begin{bmatrix} 4 & 0 & -5 & 7 \\ -3 & 8 & -2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} -11 & 3 & -1 & 6 \\ 5 & 2.5 & 0 & -6 \end{bmatrix} \quad C = \begin{bmatrix} 0 & -8 & 13 & -5 \\ 1 & -4 & 7.5 & 9 \end{bmatrix}$$

1. $3A$  2. $A + B$  3. $C - A$  4. $2A + 2C$
9. $C - B + A$  10. $5A - 2B$  11. $2(A + C)$  12. $\frac{2}{3}C$
13. $-\frac{2}{5}B$  14. $3A + 2B - 4C$  15. $(A + 2B) - 2(B + A)$  16. $-10B + 20C$
17. element $A_{12}$  18. element $B_{24}$  19. dimensions of $C$  20. $\frac{1}{2}$ (element $C_{21}$)

Solve for $x$ and $y$.

21. \[ \begin{bmatrix} x + y \\ x - y \end{bmatrix} = \begin{bmatrix} -2 \\ -8 \end{bmatrix} \]
22. \[ \begin{bmatrix} 3x + 2y \\ x - 5y \end{bmatrix} = \begin{bmatrix} -4 \\ 27 \end{bmatrix} \]
23. \[ \begin{bmatrix} 15 - 6x \\ 8x + 3y \end{bmatrix} = \begin{bmatrix} 3y \\ 13 \end{bmatrix} \]

24. $[3x \quad -2.5y] = [51 \quad 40]$
25. $[x + 2y \quad 3x - 5y] = [-1 \quad 2.5]$
26. \[ \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} + 3 \end{bmatrix} = \begin{bmatrix} -\frac{3}{4} \\ -\frac{13}{52} \end{bmatrix} \]

Use the given matrices to find each of the following. If not possible, write NP.

$$M = \begin{bmatrix} 4 & -3 \\ 2 & 1 \end{bmatrix} \quad N = \begin{bmatrix} 0 & 2 \\ -5 & 4 \end{bmatrix} \quad P = \begin{bmatrix} 6 & 0 & 10 \\ -1 & 1 & 7 \end{bmatrix} \quad Q = \begin{bmatrix} -8 & 2 \\ 1 & 9 \\ -3 & 5 \end{bmatrix} \quad R = \begin{bmatrix} -6 \\ 10 \end{bmatrix}$$

27. $MN$  28. $MP$  29. $MQ$  30. $MR$
31. $NM$  32. $PM$  33. $QM$  34. $RM$
35. $NP$  36. $NQ$  37. $NR$  38. $PN$
39. $QN$  40. $RN$  41. $PQ$  42. $PR$
43. $QP$  44. $RP$  45. $QR$  46. $RQ$
47. $MN + PQ$  48. $MR - NR$  49. $M^2$  50. $N^3$
51. $NMP$  52. $PQN$  53. $MPQ$  54. $-QPQ$
55. $QNR$  56. $R(M + N)$  57. $(M + N)R$  58. $QMP + \frac{1}{2}QP$

Solve for $x$ and $y$.

59. \[ \begin{bmatrix} 6 \\ -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -8 \\ -22 \end{bmatrix} \]
60. \[ \begin{bmatrix} 14 \\ -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -7 \end{bmatrix} \]
Trapezoid $ABCD$ has vertices $A(-5, 0)$, $B(-5, 3)$, $C(-2, 4)$, and $D(1, 2)$. Graph the trapezoid and its image after each of the following rotations about the origin from the original position. (Lesson 8-2)

61. $90^\circ$ counterclockwise
62. $180^\circ$ clockwise
63. $45^\circ$ clockwise

The following sets of points are the vertices of $PQRS$ and its dilation image. Name the scale factor and the center of dilation. (Lesson 8-3)

64. $P(-2, 2)$, $Q(-2, 5)$, $R(-3, 5)$, $S(-6, 2)$
   $P'(-2, 2)$, $Q'(-2, 20)$, $R'(-8, 20)$, $S'(-26, 2)$

Tell whether the order in which you perform each pair of transformations affects the image produced. If it does affect the image, sketch an example. (Lesson 8-4)

65. rotation and translation
66. dilation (center at figure vertex) and translation
67. dilation (center at the origin) and reflection
68. rotation and reflection

### Career – Aerospace Engineer

Aerospace engineers use their knowledge of structural design, aerodynamics, propulsion, thermodynamics, and acoustics to design roller coasters. They know how tight a turn can be without endangering passengers or damaging the coaster’s structure. These workers use science to make a roller coaster fast, fun and safe.

Aerospace engineers are also employed to build aircraft and spacecraft and to develop military technology. These engineers apply technology learned in other industries to transportation on land, sea and air. Aerospace engineers must understand math and physics and how to use computers, calculators and other tools to test their ideas.

You have designed a roller coaster in the shape shown at the right. This coaster is for people over 48 in. tall. The amusement park now wants you to design a children’s version of the coaster on a smaller scale with a less steep first hill.

1. Find the slope between points $A$ and $B$.
2. Find the slope between points $B$ and $C$.
3. Replot point $B$ so that the slope of $AB$ is $2$ and slope of $BC$ is $-2$. What are the new coordinates for $B$?

[Diagram of roller coaster]
Work with a partner. Use the point (2, 6) to answer each of the following.

1. What point is (2, 6) reflected over the x-axis?
2. What point is (2, 6) reflected over the y-axis?
3. What point is (2, 6) reflected over the line $y = x$?
4. What point is (2, 6) reflected over the line $y = -x$?

### BUILD UNDERSTANDING

A point can be represented by a matrix, as well as an ordered pair. The ordered pair (2, 6) can be represented by the matrix $\begin{bmatrix} 2 \\ 6 \end{bmatrix}$.

The element in the first row is the x-coordinate, and the element in the second row is the y-coordinate.

In general, the ordered pair $(x, y)$ is represented by the matrix $\begin{bmatrix} x \\ y \end{bmatrix}$.

In a similar way, a matrix can be used to denote a polygon. Because each vertex is a point, each point can be represented by a matrix. These four matrices for the vertices can be combined into a single matrix.

### Example 1

First, represent each vertex of quadrilateral $ABCD$ with a $2 \times 1$ matrix. Then combine these matrices into a single $2 \times 4$ matrix.

#### Solution

The vertices can be represented as follows.

$\begin{bmatrix} -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \end{bmatrix}, \begin{bmatrix} 6 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

Putting the column matrices into a single $2 \times 4$ matrix, you have the following.

$\begin{bmatrix} -1 & 4 & 6 & 1 \\ 3 & 6 & 2 & -2 \end{bmatrix}$

Each column refers to one vertex of the quadrilateral.
Just as points and polygons can be represented by matrices, you can represent transformations with matrices. Below is a table of matrices for reflections.

<table>
<thead>
<tr>
<th>Reflection</th>
<th>Matrix</th>
<th>Reflection</th>
<th>Matrix</th>
</tr>
</thead>
</table>
| over the x-axis     | \[
\begin{bmatrix} 1 & 0 \\
0 & -1 \\
\end{bmatrix}
\] | over the line \( y = x \) | \[
\begin{bmatrix} 0 & 1 \\
1 & 0 \\
\end{bmatrix}
\] |
| over the y-axis     | \[
\begin{bmatrix} -1 & 0 \\
0 & 1 \\
\end{bmatrix}
\] | over the line \( y = -x \) | \[
\begin{bmatrix} 0 & -1 \\
-1 & 0 \\
\end{bmatrix}
\] |

**Example 2**

Find the reflection image of \( \triangle ABC \) with vertices at \( A(1, -2) \), \( B(6, -2) \), and \( C(4, -5) \) when the triangle is reflected over the line \( y = x \). Use matrices.

**Solution**

Triangle \( ABC \) can be represented by 
\[
\begin{bmatrix}
1 & 6 & 4 \\
-2 & -2 & -5
\end{bmatrix}
\]

The matrix representing a reflection over the line \( y = x \) is 
\[
\begin{bmatrix} 0 & 1 \\
1 & 0 \\
\end{bmatrix}
\]

Multiply the two matrices. 
\[
\begin{bmatrix} 0 & 1 \\
1 & 0 \\
\end{bmatrix} \begin{bmatrix} 1 & 6 & 4 \\
-2 & -2 & -5
\end{bmatrix} = \begin{bmatrix} -2 & -2 & -5 \\
1 & 6 & 4
\end{bmatrix}
\]

So, the image of \( \triangle ABC \) is 
\[
\begin{bmatrix} -2 & -2 & -5 \\
1 & 6 & 4
\end{bmatrix}
\]

**Example 3**

**GRAPHIC ART** An artist is creating a border using design software. To create the basic pattern, she enters the coordinates for \( \triangle XYZ \) with vertices \( X(0, 0) \), \( Y(2, -3) \), and \( Z(6, -3) \). She wants to draw the triangle reflected over the y-axis. Find the coordinates of the reflection image using matrices.

**Solution**

Let 
\[
\begin{bmatrix} 0 & 2 & 6 \\
0 & -3 & -3
\end{bmatrix}
\]
represent the triangle.

Then multiply by 
\[
\begin{bmatrix} -1 & 0 \\
0 & 1 \\
\end{bmatrix}
\]
the matrix for a reflection over the y-axis.

\[
\begin{bmatrix} -1 & 0 \\
0 & 1 \\
\end{bmatrix} \begin{bmatrix} 0 & 2 & 6 \\
0 & -3 & -3
\end{bmatrix} = \begin{bmatrix} 0 & -2 & -6 \\
0 & -3 & -3
\end{bmatrix}
\]

So, the coordinates of the \( X'Y'Z' \) are shown in the matrix 
\[
\begin{bmatrix} 0 & -2 & -6 \\
0 & -3 & -3
\end{bmatrix}
\]
Try These Exercises

Represent each geometric figure with a matrix.

1. 

2. 

3. 

RIDE DESIGN  Amusement park rides are tested using computer simulations. A triangle with vertices \( \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ 7 \\ 1 \end{pmatrix}, \begin{pmatrix} 7 \\ 3 \\ 2 \end{pmatrix} \) is used to represent a moving platform to which the ride's cars are attached. Find the reflection images of the triangle.

4. over the \( x \)-axis.  5. over the line \( y = x \).  6. over the \( y \)-axis.  7. over the line \( y = -x \).

Practice Exercises • For Extra Practice, see page 689.

Represent each geometric figure with a matrix.

8. 

9. 

10. 

Calculator  Multiply matrices using a graphing calculator to find the following reflection images of the quadrilateral represented by the matrix \( \begin{pmatrix} -2 & 1 & 7 & 4 \\ 7 & 3 & 4 & 7 \end{pmatrix} \).

11. over the line \( y = x \)  12. over the \( y \)-axis  13. over the line \( y = -x \)  14. over the \( x \)-axis

Interpret each equation as indicating:

The reflection image of point \( \_ \_ ? \_ \_ \) over \( \_ \_ ? \_ \_ \) is the point \( \_ \_ ? \_ \_ \).

15. \( \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 4 \\ 4 \end{pmatrix} = \begin{pmatrix} -4 \\ -4 \end{pmatrix} \)

16. \( \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \end{pmatrix} \)

17. \( \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -2 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \end{pmatrix} \)

18. \( \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 5 \\ -3 \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} \)

Talk About It  Toni says that you can produce any dilation with center at the origin and a scale factor of \( k \) using the matrix \( \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix} \). Does Toni's thinking make sense?
GAME DEVELOPMENT In a hand-held game, the player must click on falling stars within a time limit. To develop the game, the programmer specifies the coordinates for a star to appear. Two seconds later, the image of the star appears under a reflection. Exercises 20–23 specify image and preimage points used in the game. For each, name the reflecting line and verify your answer by matrix multiplication.

20. preimage (5, –1), image (–1, 5)  21. preimage (2, 0), image (0, –2)
22. preimage (b, a), image (b, –a)  23. preimage (7, 3), image (–7, 3)

24. GEOMETRY SOFTWARE Find the image of rhombus $ABCD$ under the transformation associated with matrix $M$. Graph both the preimage and its image using geometric-drawing software.

$$M = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \quad ABCD = \begin{bmatrix} 3 & 5 & 3 & 1 \\ 2 & 0 & –2 & 0 \end{bmatrix}$$

EXTENDED PRACTICE EXERCISES

25. WRITING MATH What type of transformation is represented by the matrix $\begin{bmatrix} –2 & 0 \\ 0 & –2 \end{bmatrix}$?

26. Draw any triangle in the coordinate plane. Represent it with a $2 \times 3$ matrix. Then apply each transformation below. Draw each preimage and image on a coordinate grid.

a. $\begin{bmatrix} 0 & –1 \\ 1 & 0 \end{bmatrix}$  b. $\begin{bmatrix} –1 & 0 \\ 0 & –1 \end{bmatrix}$  c. $\begin{bmatrix} 0 & 1 \\ –1 & 0 \end{bmatrix}$

27. Refer to the graphs you drew in Exercise 26.

a. Which shows a clockwise rotation of 90°?

b. Which shows a clockwise rotation of 180°?

c. Which shows a counterclockwise rotation of 90°?

28. CHAPTER INVESTIGATION Measure the angle of descent for each hill in your roller coaster design. Estimate your coaster’s top speed by comparing its features to the coasters shown in the table on page 337. Write a paragraph to justify your estimate.

MIXED REVIEW EXERCISES

Find the slope and $y$-intercept for each line. (Lesson 6-1)

29. $y = \frac{1}{2}x – 3$  30. $y = –3x + 4$
31. $2y – x = 6$  32. $3(x – 4) = 5y$
33. $3y – 4x – 7 = 0$  34. $2x = 4y + 2$
Some problems can be solved by translating directly to a matrix and then performing matrix operations. Consider using matrix operations to solve problems whenever information can be easily organized into tables with corresponding elements.

**Problem**

**BUSINESS** An orchard grows Delicious, Jonathan, and Granny Smith apples. The apples are sold in boxes to two different markets. The profit is $5.85 on a box of Delicious apples, $4.25 on Jonathans, and $3.75 on Granny Smiths. The table shows the number of boxes sold.

Find the amount of profit generated by sales to each market.

<table>
<thead>
<tr>
<th>Apples</th>
<th>Markets</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bill’s</td>
</tr>
<tr>
<td>Delicious</td>
<td>250</td>
</tr>
<tr>
<td>Jonathan</td>
<td>320</td>
</tr>
<tr>
<td>Granny Smith</td>
<td>175</td>
</tr>
</tbody>
</table>

**Solve the Problem**

a. Represent the market data in a $3 \times 2$ matrix as shown.

$$A = \begin{bmatrix} 250 & 225 \\ 320 & 295 \\ 175 & 190 \end{bmatrix}$$

b. Write a matrix to represent the respective profits. Think about the dimensions necessary for matrix multiplication. Since matrix $A$ has three rows, matrix $B$ must have three columns. The dimensions of $B$ must be $1 \times 3$.

$$B = \begin{bmatrix} 5.85 & 4.25 & 3.75 \end{bmatrix}$$

c. The product $BA$ will be a $1 \times 2$ matrix that determines the profit from each market.

$$BA = \begin{bmatrix} 5.85 & 4.25 & 3.75 \end{bmatrix} \begin{bmatrix} 250 & 225 \\ 320 & 295 \\ 175 & 190 \end{bmatrix} = \begin{bmatrix} 3478.75 & 3282.50 \end{bmatrix}$$

The profit from Bill’s market is $3478.75. The profit from Jan’s market is $3282.50.
TRY THESE EXERCISES

Use matrices to solve each problem.

1. **FOOD DISTRIBUTION** A farm raises two crops, which are shipped to three distributors. The table shows the number of crates shipped to distributors.

   The profit on crop 1 is $2.75 per crate. The profit on crop 2 is $3.20 per crate.

   Find the amount of profit from each distributor.

<table>
<thead>
<tr>
<th>Distributor</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crop 1</td>
<td>350</td>
<td>275</td>
<td>550</td>
</tr>
<tr>
<td>Crop 2</td>
<td>200</td>
<td>310</td>
<td>260</td>
</tr>
</tbody>
</table>

2. **FOOD CONCESSIONS** A large amusement park owns four bakeries, each of which produces three types of bread: white, rye and whole wheat. The bread is used to supply food vendors throughout the park. The number of loaves produced daily at each bakery is shown in the table at the right.

   By baking its own bread, the park can reduce the amount of overhead and increase its profits. The profit on each loaf of bread is 75 cents for white, 50 cents for rye, and 60 cents for whole wheat. Find the amount of profit from each bakery.

<table>
<thead>
<tr>
<th>Bakery</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>White</td>
<td>190</td>
<td>215</td>
<td>240</td>
<td>112</td>
</tr>
<tr>
<td>Rye</td>
<td>65</td>
<td>80</td>
<td>110</td>
<td>60</td>
</tr>
<tr>
<td>Whole wheat</td>
<td>205</td>
<td>265</td>
<td>290</td>
<td>170</td>
</tr>
</tbody>
</table>

3. **SALES** A sneaker manufacturer makes five kinds of sneakers: basketball, running, walking, cross-trainer, and tennis. The sneakers are shipped to three retail outlets. The number of pairs of sneakers shipped to each outlet is shown.

   Profit on each pair of sneakers is as follows:

   - basketball $4.50
   - running $3.50
   - walking $3.75
   - cross-trainer $5.25
   - tennis $5.00

   Find the amount of profit for each outlet.

<table>
<thead>
<tr>
<th>Outlets</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basketball</td>
<td>30</td>
<td>40</td>
<td>35</td>
</tr>
<tr>
<td>Running</td>
<td>20</td>
<td>25</td>
<td>30</td>
</tr>
<tr>
<td>Walking</td>
<td>15</td>
<td>20</td>
<td>25</td>
</tr>
<tr>
<td>Cross-trainer</td>
<td>15</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>Tennis</td>
<td>25</td>
<td>30</td>
<td>25</td>
</tr>
</tbody>
</table>

4. **SOUVENIRS** An amusement park sells hats, T-shirts and stuffed toys. The table on the left gives the number of each type of souvenir sold during a two-week period. The table on the right gives the price of each souvenir. Find the total amount spent on souvenirs each week.

<table>
<thead>
<tr>
<th>Item</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hats</td>
<td>$14</td>
</tr>
<tr>
<td>T-shirts</td>
<td>$18</td>
</tr>
<tr>
<td>Toys</td>
<td>$24</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Week 1</th>
<th>Hats</th>
<th>T-shirts</th>
<th>Toys</th>
</tr>
</thead>
<tbody>
<tr>
<td>128</td>
<td>240</td>
<td>58</td>
<td></td>
</tr>
<tr>
<td>Week 2</td>
<td>130</td>
<td>215</td>
<td>89</td>
</tr>
</tbody>
</table>

MIXED REVIEW EXERCISES

Find the sum of the measures of the angles of a convex polygon with the given number of sides. (Lesson 4-7)

5. 37
6. 52
7. 29
8. 45
9. 62
10. 40
11. 58
12. 19
Chapter 8 Review

VOCABULARY

Choose the word from the list that best completes each statement.

1. A translation, reflection, rotation, or dilation is known as a ___?__ of a figure.
2. A dilation image is obtained by multiplying the length of each side of a figure by a number called the ___?__.
3. Under a transformation, the new figure is called the image and the original figure is called the ___?__.
4. The number of rows and columns are the ___?__ of the matrix.
5. A ___?__ is a rectangular array of numbers arranged into rows and columns.
6. Sliding a figure is called a ___?__.
7. When a matrix is multiplied by a number, the number is called a ___?__.
8. Turning a figure around a point is called a ___?__.
9. Flipping a figure over a line is called a ___?__.
10. Reducing or enlarging a figure is called a ___?__.

LESSON 8-1 Translations and Reflections, p. 338

Under a translation, an image is produced by sliding every point of the original figure the same distance in the same direction.

Under a reflection, a figure is flipped over a line of reflection.

Copy each graph on graph paper. Then draw the image of each figure under the given translation.

11. 4 units up
12. 3 units left

Copy each graph on graph paper. Then draw the image of each figure under the given reflection.

13. over the x-axis
14. over the y-axis
LESSON 8-2  Rotations in the Coordinate Plane, p. 342

Under a rotation, a figure is turned about a point.

Copy each graph on graph paper. Then draw the image of each figure under the given rotation about the origin.

15. 90° counterclockwise

![Graph showing 90° counterclockwise rotation]

16. 180° clockwise

![Graph showing 180° clockwise rotation]

LESSON 8-3  Dilations in the Coordinate Plane, p. 348

A dilation is a transformation that produces an image of the same shape, but a different size.

Copy each graph on graph paper. Then draw the image of each figure under the given dilation with the center at the origin.

17. scale factor of \(\frac{1}{3}\)

![Graph showing dilation with scale factor \(\frac{1}{3}\)]

18. scale factor of 2

![Graph showing dilation with scale factor 2]

LESSON 8-4  Multiple Transformations, p. 352

Two or more successive transformations can be applied to a given figure. This is called a composite of transformations.

Describe two transformations that would create the image in blue. There may be more than one possible answer.

19.

![Graph showing transformations]

20.

![Graph showing transformations]

LESSON 8-5  Addition and Multiplication with Matrices, p. 358

A matrix is a rectangular array of numbers arranged into rows and columns. The number of rows and columns are the dimensions of the matrix. The numbers that make up the matrix are the elements of the matrix.
21. Find the dimensions of \( D \).
\[
D = \begin{bmatrix}
1 & -2 & 3 & 2 \\
-4 & 5 & -6 & 4 \\
7 & -8 & 9 & 6
\end{bmatrix}
\]

22. Identify the elements \( D_{32}, D_{21}, \) and \( D_{13} \).

23. Find \( kD \) for \( k = -2 \).

Use matrices \( A - C \) to find each of the following.
\[
A = \begin{bmatrix}
-3 & 5 & 0 & 2 \\
1 & -7 & 6 & -1
\end{bmatrix} \quad B = \begin{bmatrix}
0 & 1 & -3 & 11 \\
13 & 2 & -4 & 1
\end{bmatrix} \quad C = \begin{bmatrix}
6 & 0 & 8 & -2 \\
0 & 15 & -1 & 5
\end{bmatrix}
\]

24. \( A + B - C \)

25. \( C - B - A \)

26. \( 3(A + C) \)

**LESSON 8-6 More Operations on Matrices, p. 362**

The product of an \( m \times n \) matrix and an \( n \times p \) matrix is an \( m \times p \) matrix.

Find each product. If not possible, write NP.

27. \[
\begin{bmatrix}
3 & -2 & -4 \\
2 & 1 & 3
\end{bmatrix}
\begin{bmatrix}
0 & 1 \\
2 & -2 \\
6 & 5
\end{bmatrix}
\]

28. \[
\begin{bmatrix}
5 & -2 & -1 \\
8 & 0 & 3
\end{bmatrix}
\begin{bmatrix}
-4 & 2 \\
1 & 0
\end{bmatrix}
\]

**LESSON 8-7 Transformations and Matrices, p. 368**

The point represented by \((x, y)\) can also be represented by the matrix \( \begin{bmatrix} x \\ y \end{bmatrix} \).

Polygons and transformations can also be represented by matrices.

29. Triangle \( DEF \) is represented by \( \begin{bmatrix} 4 & 6 & 5 \\ 2 & 1 & 4 \end{bmatrix} \). Use the matrix \( \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \) to find the image when \( \triangle DEF \) is reflected over the \( y \)-axis.

30. Quadrilateral \( PQRS \) is represented by \( \begin{bmatrix} 1 & 2 & 5 & 3 \\ 1 & 3 & 2 & -1 \end{bmatrix} \). Use the matrix \( \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \) to find the image when the quadrilateral is reflected over the line \( y = -x \).

**LESSON 8-8 Problem Solving Skills: Use a Matrix, p. 372**

Some problems can be solved by translating to a matrix and using matrix operations.

31. The table at the right shows the number of students in a school’s beginning and advanced orchestra classes. The students pay a fee for instruction books: $5 for brass, $5 for woodwinds, and $8 for strings. Find the amount each class will spend on books.

<table>
<thead>
<tr>
<th></th>
<th>Advanced</th>
<th>Beginning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brass</td>
<td>12</td>
<td>28</td>
</tr>
<tr>
<td>Strings</td>
<td>22</td>
<td>25</td>
</tr>
<tr>
<td>Woodwinds</td>
<td>18</td>
<td>30</td>
</tr>
</tbody>
</table>

**CHAPTER INVESTIGATION**

**EXTENSION** Make a presentation to your class of your roller coaster. Explain why you designed the ride as you did. Display and use your marketing brochure during your presentation.
Chapter 8 Assessment

Use the figure at the right for Exercises 1–3.

1. Graph the image of \( \triangle ABC \) under a translation 5 units to the right. Label the image points \( A', B', \) and \( C' \).

2. Graph the image of \( \triangle ABC \) under a reflection across the \( x \)-axis. Label the image points \( A'', B'', \) and \( C'' \).

3. Graph the image of \( \triangle ABC \) under a 180° clockwise rotation about the origin. Label the image points \( A''', B''', \) and \( C''' \).

4. Draw the dilation image of rectangle \( LIMB \) with the center of dilation at the origin and a scale factor of \( \frac{1}{3} \).

5. Describe two transformations that together could have been used to create the image shown in blue.

Use matrix \( T \) at the right for Exercises 6–8.

6. Find the dimensions of \( T \).

7. Identify the elements \( T_{32}, T_{23}, \) and \( T_{11} \).

8. Find \( kT \) for \( k = -3 \).

9. Find the product \( MN \): \( M = \begin{bmatrix} 5 & 1 & -2 \\ 3 & 6 & 1 \end{bmatrix} \quad N = \begin{bmatrix} 4 & 3 \\ 8 & -1 \\ 0 & 7 \end{bmatrix} \)

10. Find the image of the rectangle with vertices \( \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}, \begin{bmatrix} -5 \\ -5 \\ -5 \end{bmatrix}, \begin{bmatrix} -2 \\ -2 \\ -2 \end{bmatrix} \), and \( \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix} \) under the transformation represented by \( \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \).

11. A quilt maker has three retail outlets where quilts and pillows are sold. The table shows the number of quilts and pillows sold at each outlet. Find the profit from each outlet. Use matrices.

The profit from each quilt is $90 and the profit from each pillow is $25.

<table>
<thead>
<tr>
<th>Item</th>
<th>Outlet A</th>
<th>Outlet B</th>
<th>Outlet C</th>
</tr>
</thead>
<tbody>
<tr>
<td>quilts</td>
<td>21</td>
<td>20</td>
<td>28</td>
</tr>
<tr>
<td>pillows</td>
<td>30</td>
<td>19</td>
<td>27</td>
</tr>
</tbody>
</table>


6. The graph of which equation has the greatest slope? (Lesson 6-1)
   A. $2x - 5y = 7$
   B. $4x - 5y = 10$
   C. $4y - 7x = 15$
   D. $3y - 4x = 6$

7. What is the value of $y$ for the following system of equations? (Lessons 6-5, 6-6, and 6-7)
   $7x - 2y = 22$
   $3x + y = 15$
   A. 2
   B. 3
   C. 4
   D. 5

8. In the figure below, $MN \parallel YZ$. Find the value of $a$. (Lesson 7-6)
   A. $2\frac{2}{3}$
   B. 6
   C. 10
   D. 13.5

9. Which transformation is a reflection? (Lesson 8-1)
   A.   
   B.   
   C.   
   D.  

10. What are the dimensions of the following product? (Lesson 8-6)
    $\begin{bmatrix} 10 & 2 & 0 \\ 1 & 3 & 5 \end{bmatrix} \begin{bmatrix} 6 & 4 \\ 0 & -1 \\ 3 & 2 \end{bmatrix}$
    A. $2 \times 2$
    B. $2 \times 3$
    C. $3 \times 2$
    D. $3 \times 3$
Test-Taking Tip

Question 20
Always check your work for careless errors. To check your answer to this question, remember the following rules. To translate a figure 7 units down, add \(-7\) to the \(y\)-coordinate. To reflect a figure across the \(y\)-axis, change the sign of the \(x\)-coordinate. To rotate a figure 180° clockwise, change the sign of both coordinates. To dilate a figure, multiply each coordinate by the scale factor.